Contents

Preface v

1 Mathematical Preliminaries 1

1.1 Properties of Analytic Functions 1
Problems for Section 1.1 18
1.2 Hilbert Transforms 22
Problems for Section 1.2 27
1.3 Riemann–Hilbert Problems 29
1.3.1 Some Definitions 30
1.3.2 The Index of a Function 31
1.3.3 Homogeneous Problem 35
1.3.4 Solution of the Non-Homogeneous Problem 38
Problems for Section 1.3 39
1.4 Fourier Transforms 42
Problems for Section 1.4 52
1.5 Laplace Transforms 55
Problems for Section 1.5 56
1.6 Fourier Series 56
Problems for Section 1.6 62
1.7 Transformations of Functions 65
Problems for Section 1.7 70
1.8 Spaces of Analytic Functions 74
1.8.1 Functions Analytic on the Unit Disc 74
1.8.2 Functions Analytic on Ω_+ 77
1.8.3 Functions Analytic in the Strip $\mathcal{D}_d$ 80
Problems for Section 1.8 84
1.9 The Paley–Wiener Theorem 87
Problems for Section 1.9 90
1.10 The Cardinal Function 91
Problems for Section 1.10 97
Historical Remarks on Chapter 1 102

2 Polynomial Approximation **105**
2.1 Chebyshev Polynomials 105
Problems for Section 2.1 108
2.2 Discrete Fourier Polynomials 110
Problems for Section 2.2 115
2.3 The Lagrange Polynomial 118
Problems for Section 2.3 122
2.4 Faber Polynomials 127
Problems for Section 2.4 129
Historical Remarks on Chapter 2 130

3 Sinc Approximation in Strip **131**
3.1 Sinc Approximation in $\mathcal{D}_d$ 131
Problems for Section 3.1 141
3.2 Sinc Quadrature on $(-\infty, \infty)$ 143
Problems for Section 3.2 146
3.3 Discrete Fourier Transforms on $(-\infty, \infty)$ 147
Problems for Section 3.3 151
3.4 Cauchy-Like Transforms on $(-\infty, \infty)$ 152
Problems for Section 3.4 160
3.5 Approximation of Derivatives in $\mathcal{D}_d$ 162
Problems for Section 3.5 166
3.6 The Indefinite Integral on $(-\infty, \infty)$ 167
Problems for Section 3.6 177
Historical Remarks on Chapter 3 178

4 Sinc Approximation on Γ **179**
4.1 Basic Definitions 179
Problems for Section 4.1 182
4.2 Interpolation and Quadrature on Γ 183
Problems for Section 4.2 197
4.3 Hilbert and Related Transforms on Γ 201
Problems for Section 4.3 204
4.4 Approximation of Derivatives on Γ 207
Problems for Section 4.4 217
4.5 Indefinite Integral Over Γ 218
Problems for Section 4.5 220
4.6 Indefinite Convolution Over Γ 221
4.6.1 Approximation Procedure 222
4.6.2 Formula Derivation 225
4.6.3 Convergence 228
4.6.4 Applications 234
Problems for Section 4.6 240
Historical Remarks on Chapter 4 240

5 Sinc-Related Methods **243**
5.1 Introduction 243
5.2 Variations of the Sinc Basis 245
Problems for Section 5.2 252
5.3 Elliptic Function Interpolants 252
Problems for Section 5.3 257
5.4 Inversion of the Laplace Transform 258
Problems for Section 5.4 261
5.5 Sinc-Like Rational Approximation 262
Problems for Section 5.5 276
5.6 Rationals and Extrapolation 276
5.6.1 Padé Approximation 277
5.6.2 Continued Fractions 280
5.6.3 The Epsilon Algorithm and Aitken's Δ^2 Process . . 283
5.6.4 Chebyshev Acceleration 284
5.6.5 Thiele's Algorithm 285
Problems for Section 5.6 291
5.7 Heaviside and Filter Rationals 294
5.7.1 Approximation of the Heaviside Function 295
5.7.2 Approximation of the Filter Function 298
5.7.3 Approximation of the Delta Function 299
Problems for Section 5.7 302
5.8 Positive Base Approximation 304
Problems for Section 5.8 310
Historical Remarks on Chapter 5 310

6 Integral Equations **311**
6.1 Introduction 311
6.2 Mathematical Preliminaries 313
6.2.1 Use of Functional Analysis 313
6.2.2 Approximation, Convergence, and Error 317
6.2.3 Fredholm Alternative 320
6.2.4 Perturbed Equations 325
6.2.5 Tikhonov Regularization 326
Problems for Section 6.2 329
6.3 Reduction to Algebraic Equations 331
6.3.1 Galerkin Method 331
6.3.2 Nyström's Method 331
6.3.3 The Generalized Inverse Procedure 332
6.3.4 Errors in the Numerical Solution 335
Problems for Section 6.3 336
6.4 Volterra Integral Equations 338
6.4.1 Linear Volterra Equations 338
6.4.2 Non-Linear Equations via Neumann Series 340
6.4.3 Non-Linear Equations by Newton's Method 343

Problems for Section 6.4 346
6.5 Potential Theory Problems 349
6.5.1 Problem Description 349
6.5.2 Spaces for Sinc Approximation 350
6.5.3 Sinc Approximation 351
6.5.4 Properties of the Integral Equation 354
6.5.5 Galerkin Approximation 360
6.5.6 Several Surface Patches—Domain Decomposition . . 361
6.5.7 An Explicit Example 364
6.5.8 Kernel Singularities and Integration 374
Problems for Section 6.5 376
6.6 Reduced Wave Equation on a Half-Space 379
6.6.1 Problem Description 380
6.6.2 Spaces for Sinc Approximation 381
6.6.3 Sinc Approximation 383
6.6.4 Properties of the Integral Equation 385
6.6.5 Galerkin Approximation 391
6.6.6 Evaluation of Moment Integrals 392
6.6.7 Numerical Evaluation of Solution 395
6.6.8 An Explicit Example 396
Problems for Section 6.6 398
6.7 Cauchy Singular Integral Equations 400
6.7.1 The Problem 401
6.7.2 The Method of Regularization 401
6.7.3 Properties of the Fredholm Equation 404
6.7.4 Approximation via Nyström's Method 406
Problems for Section 6.7 407
6.8 Convolution-Type Equations 407
6.8.1 The Problems and Theoretical Solutions 407
6.8.2 Approximate Solution 410
6.8.3 Explicit Examples 418
Problems for Section 6.8 421
6.9 The Laplace Transform and Its Inversion 422
Problems for Section 6.9 437
Historical Remarks on Chapter 6 439

7 Differential Equations **441**
7.1 ODE–IVP 444
7.1.1 Linear Initial Value Problems 445
7.1.2 Non-Linear Initial Value Problems 453
Problems for Section 7.1 459
7.2 ODE–BVP 465
7.2.1 Sinc–Galerkin and Collocation 467
7.2.2 Integration by Parts 469
7.2.3 Collocation and Integration by Parts 470

7.2.4 Convergence of Sinc–Galerkin Methods 471
7.2.5 Non-Linear Equations 483
7.2.6 Non-Homogeneous Boundary Conditions 484
7.2.7 Symmetric Sinc–Galerkin Method 486
7.2.8 More Examples 492
Problems for Section 7.2 497
7.3 Analytic Solutions of PDE 500
7.3.1 Analyticity of Solutions in All Variables 501
7.3.2 Analyticity for Sinc Approximation 501
7.3.3 Singularities Due to Corners and Edges 506
Problems for Section 7.3 509
7.4 Elliptic Problems 511
Problems for Section 7.4 521
7.5 Hyperbolic Problems 521
Problems for Section 7.5 523
7.6 Parabolic Problems 524
Problems for Section 7.6 529
Historical Remarks on Chapter 7 529

References **533**

Index **563**