



# Table of Contents

Introduction . . . . .	I
1. Discrete Iterations and Automata Networks: Basic Concepts . . . . .	1
1. Discrete Iterations and Their Graphs . . . . .	1
2. Examples . . . . .	3
3. Connectivity Graphs and Incidence Matrices . . . . .	7
4. Interpretations in Terms of Automata Networks . . . . .	9
5. Serial Operation and the Gauss-Seidel Operator . . . . .	11
6. Serial-Parallel Modes of Operation and the Associated Operators	17
2. A Metric Tool . . . . .	27
1. The Boolean Vector Distance $d$ . . . . .	28
2. Some Basic Inequalities . . . . .	29
3. First Applications . . . . .	31
4. Serial-Parallel Operators. An Outline . . . . .	36
5. Other Possible Metric Tools . . . . .	40
3. The Boolean Perron-Frobenius and Stein-Rosenberg Theorems . . . . .	43
1. Eigenelements of a Boolean Matrix . . . . .	43
2. The Boolean Perron-Frobenius Theorem . . . . .	51
3. The Boolean Stein-Rosenberg Theorems . . . . .	53
4. Conclusion . . . . .	55
4. Boolean Contraction and Applications . . . . .	57
1. Boolean Contraction . . . . .	58
2. A Fixed Point Theorem . . . . .	58

3. Examples . . . . .	62
4. Serial Mode: Gauss-Seidel Iteration for a Contracting Operator	65
5. Examples . . . . .	66
6. Comparison of Operating Modes for a Contracting Operator . .	69
7. Examples . . . . .	70
8. Rounding-Off: Successive Gauss-Seidelisations . . . . .	76
9. Conclusions . . . . .	78
5. Comparison of Operating Modes . . . . .	79
1. Comparison of Serial and Parallel Operating Modes . . . . .	80
2. Examples . . . . .	88
3. Extension to the Comparison of Two Serial-Parallel Modes of Operation . . . . .	90
4. Examples . . . . .	91
5. Conclusions . . . . .	92
6. The Discrete Derivative and Local Convergence . . . . .	95
1. The Discrete Derivative . . . . .	95
2. The Discrete Derivative and the Vector Distance . . . . .	99
3. Application: Characterization of the Local Convergence in the Immediate Neighbourhood of a Fixed Point . . . . .	103
4. Interpretation in Terms of Automata Networks . . . . .	107
5. Application: Local Convergence in a Massive Neighbourhood of a Fixed Point . . . . .	108
6. Gauss-Seidel . . . . .	115
7. The Derivative of a Function Composition . . . . .	117
8. The Study of Cycles: Attractive Cycles . . . . .	121
9. Conclusions . . . . .	129
7. A Discrete Newton Method . . . . .	131
1. Context . . . . .	132
2. Two Simple Examples . . . . .	135
3. Interpretation in Terms of Automata . . . . .	137
4. The Study of Convergence: The Case of the Simplified Newton Method . . . . .	138
5. The Study of Convergence, The General Case . . . . .	145
6. The Efficiency of an Iterative Method on a Finite Set . . . . .	153
7. Numerical Experiments . . . . .	154
8. Conclusions . . . . .	165
General Conclusion . . . . .	166
Appendix 1. The Number of Maps of $\{0, 1\}^n$ into $\{0, 1\}^n$ . . . . .	167
Appendix 2. The Number of Regular $n \times n$ Matrices with Elements in $Z/p$ (p prime) . . . . .	171

Appendix 3. Some Further Examples Illustrating the Standard Newton Method in $(Z/2)^n$ and $(Z/3)^n$ . . . . .	173
Appendix 4. Continuous Iterations-Discrete Iterations . . . . .	179
Bibliography . . . . .	183
Index . . . . .	193