

Contents

Part I. Stability Analysis of Difference Schemes by the Method of Differential Approximation	1
1. Certain Properties of the Theory of Linear Differential Equations and Difference Schemes	3
1.1 Cauchy's Problem	3
1.2 One-dimensional Time-dependent Case	4
1.3 Systems of Second-order Equations	5
1.4 Basic Concepts of the Theory of Difference Schemes	5
2. The Concept of the Differential Approximation of a Difference Scheme	8
2.1 Γ -form and Π -form of the Differential Representation of a Difference Scheme	8
2.2 General Form of the Π -form	12
2.3 Γ - and Π -form of the First Differential Approximation	13
2.4 Remarks on Nonlinear Differential Equations	15
2.5 The Role of the First Differential Approximation	15
2.6 On the Correctness of Giving the Π -form as an Infinite Differential Equation	16
2.7 Differential Representations of Difference Schemes in Spaces of Generalized Functions	19
2.8 Asymptotic Expansion of the Solution of a Difference Scheme	30
2.9 On the Injective Character of the Mapping of Difference Schemes in the Set of Differential Representations	34
3. Stability Analysis of Difference Schemes with Constant Coefficients by Means of the Differential Representation	38
3.1 Absolute and Conditional Approximation	38
3.2 Lax' Equivalence Theorem	39
3.3 On the Necessary Stability Conditions for Difference Schemes	40
4. Connection Between The Stability of Difference Schemes and the Properties of Their First Differential Approximations	45
4.1 Simple Difference Schemes	45

4.2	Majorant Difference Schemes	47
4.3	Fractional-step Method	49
4.4	The Case of Multi-dimensional Schemes	51
4.5	Two-level Difference Schemes	54
4.6	Remarks on Nonlinear Equations	57
5.	Dissipative Difference Schemes for Hyperbolic Equations	58
5.1	Different Definitions of Dissipativity	58
5.2	Stability Theorem for Dissipative Schemes in the Generalized Sense	59
5.3	Stability Theorem for Dissipative Schemes in the Sense of Roshdestvenskii-Yanenko-Richtmyer	61
5.4	Stability Theorem for a Partly Dissipative Scheme	62
6.	A Means for the Construction of Difference Schemes with Higher Order of Approximation	63
6.1	Convergence Theorem	63
6.2	A Weakly Stable Difference Scheme	65
6.3	Construction of a Third-order Difference Scheme	66
6.4	Application to Nonlinear Equations	67
6.5	Application of the Method to a Boundary Value Problem	68
6.6	Stability Theorems for Dissipative Schemes	69
Part II. Investigation of the Artificial Viscosity of Difference Schemes		73
7.	<i>K</i> -property of Difference Schemes	75
7.1	Introduction	75
7.2	Definition of <i>K</i> -property	76
7.3	Simple Difference Schemes	77
7.4	Three-point Schemes	80
7.5	Necessary and Sufficient Conditions for the Strong Property <i>K</i>	83
7.6	Predictor-Corrector Scheme	84
7.7	Implicit Difference Schemes	85
7.8	Higher-order Difference Schemes	87
7.9	Application to Gas Dynamics	88
7.10	Connection Between Partly Dissipative Difference Schemes and Those with the Strong Property <i>K</i>	89
7.11	The Property $P_j^{(p, 1)}$	89
7.12	The Property $\mathcal{D}_j^{(p, 1)}$	90
8.	Investigation of Dissipation and Dispersion of Difference Schemes	91
8.1	Dissipation and Dispersion of Difference Schemes	91
8.2	Dissipation and Dispersion of Differential Approximations	93
8.3	Relative Dissipative Error and Dispersion	98
8.4	Geometrical Illustration of Dissipative and Dispersive Errors	100

8.5 Classification of Difference Schemes According to Dissipative Properties	101
8.6 Some Remarks on Using Finite Number of Terms of the Differential Approximation	102
8.7 Connection Between Dispersion, Dissipation and Errors of Difference Schemes	103
9. Application of the Method of Differential Approximation to the Investigation of the Effects of Nonlinear Transformations	107
9.1 Introduction	107
9.2 Equivalence of Difference Schemes	107
9.3 The Fluid Equations Including Gravity	109
9.4 The Equations of Gas Dynamics	111
10. Investigation of Monotonicity of Difference Schemes	115
10.1 Introduction	115
10.2 Moving Shock with Constant Velocity	115
11. Difference Schemes in an Arbitrary Curvilinear Coordinate System	118
11.1 Introduction	118
11.2 Definition of a Mesh	119
11.3 Closeness of Solutions of Difference Schemes on Different Meshes	120
11.4 Example of Convective Equation	122
Part III. Invariant Difference Schemes	127
12. Some Basic Concepts of the Theory of Group Properties of Differential Equations	131
12.1 Infinitesimal Operator of G_r	131
12.2 Invariant Subsets of G_r	132
12.3 Necessary and Sufficient Conditions for the Invariance of the First-order Differential Equations	133
13. Groups Admitted by the System of the Equations of Gas Dynamics	134
13.1 Lie-Algebra for Two-dimensional Gas Dynamics	134
13.2 One-dimensional Gas Dynamics	135
14. A Necessary and Sufficient Condition for Invariance of Difference Schemes on the Basis of the First Differential Approximation	136
15. Conditions for the Invariance of Difference Schemes for the One-dimensional Equations of Gas Dynamics	138
15.1 The Class of Two-level Difference Schemes for the Eulerian Equations of Gas Dynamics	138
15.2 Condition for Invariance for the Difference Scheme (15.1)	140

15.3	Property M of a Difference Scheme	144
15.4	Property K	146
15.5	Weak Solutions of Difference Scheme (15.1), $\alpha = \beta$	148
15.6	One-dimensional System of the Equations of Gas Dynamics in Lagrangean Coordinates	149
15.7	Polytropic Gas	153
16.	Investigation of Properties of the Artificial Viscosity of Invariant Difference Schemes for the One-dimensional Equations of Gas Dynamics	157
16.1	Γ -matrices in Eulerian Coordinates	157
16.2	Property \bar{K}	159
16.3	Polynomial Form of the Viscosity Matrix	161
16.4	Numerical Experiments for Equations of Gas Dynamics in Eulerian Coordinates	163
16.5	Damping of Oscillatory Effects	169
16.6	Γ -matrices in Lagrangean Coordinates	170
16.7	Width of Shock Smearing	174
16.8	Numerical Experiments on the Equations of Gas Dynamics in Lagrangean Coordinates	175
16.9	Conservative and Fully Conservative Schemes	181
17.	Conditions for the Invariance of Difference Schemes for the Two-dimensional Equations of Gas Dynamics	186
17.1	Two-level Class of Difference Schemes	186
17.2	Conditions for the Invariance of the Difference Scheme (17.1)	188
17.3	Theorem on Invariance	191
17.4	Property M	191
17.5	Property \bar{K}	194
17.6	Some Remarks on Stability	195
17.7	Γ -matrices	195
17.8	Numerical Experiments for the Problem of the Interaction of a Shock with Obstacles	198
17.9	Numerical Experiments for the Shallow Water Equations	203
17.10	Analysis of the Properties of the Artificial Viscosity for Difference Schemes of Two-dimensional Gas Flows	210
17.10.1	Schemes of Class J_1	210
17.10.2	Schemes of Class J_2	212
17.10.3	The Lax' Scheme	213
17.10.4	The Rusanov-Scheme	215
17.10.5	The Lax-Wendroff-Scheme	216
17.10.6	Two-step Variant of the Lax-Wendroff-Scheme	217
17.10.7	Modification of the Lax-Wendroff-Scheme	218
17.10.8	The MacCormack-Scheme	219
17.10.9	Comparison of Numerical Results	220

18. Investigation of Difference Schemes with Time-splitting Using the Theory of Groups	235
18.1 Two First-order Schemes with Time-splitting	235
18.2 Group Properties of the Schemes (18.1) and (18.2)	236
18.3 Conditions for Invariance for a Polytropic Gas	238
18.4 Comparison of Invariant and Noninvariant Schemes	239
Part IV. Appendix	247
A.1 Introduction	249
A.2 Difference Schemes for the Equation of Propagation	251
A.3 Difference Schemes for the Equations of One-dimensional Gas Dynamics	267
References	279
Subject Index	293