

CONTENTS

Chapter 1	Introduction	1
1.1	The Finite-Element Method,	1
1.2	The Mathematics of Finite Elements,	2
1.3	The Present Study,	3
1.4	Notations and Preliminaries,	4
	References,	5

PART I MATHEMATICAL FOUNDATIONS

Chapter 2	Distributions, Mollifiers, and Mean Functions	9
2.1	Introduction,	9
2.2	Functionals and Test Functions on One-Dimensional Domains,	10
2.3	Distributions,	14
2.4	Locally Integrable Generators, Regular and Singular Distributions,	17
2.5	Some Properties of Distributions,	20
2.6	Distributional Differential Equations,	26
2.7	Distributions and Generalized Functions in \mathbb{R}^n ,	31
2.8	Fourier Transforms, Rapidly Decaying Functions, and Tempered Distributions,	36
2.9	Weak and Strong Derivatives in $L_p(\Omega)$,	45
2.10	Mollifiers and Mean Functions,	46
	References,	54

Chapter 3	Theory of Sobolev Spaces	55
3.1	Introduction,	55
3.2	The Sobolev Space $W_p^m(\Omega)$,	55
3.3	Partitions of Unity, Boundaries, and Cone Conditions,	57
3.4	Some Properties of the Sobolev Spaces $W_p^m(\Omega)$ and $\dot{W}_p^m(\Omega)$,	61
3.5	The Sobolev Integral Identity,	67
3.6	The Sobolev Embedding Theorems,	79
3.7	The Decomposition of $W_p^m(\Omega)$,	82
	References,	88
Chapter 4	Hilbert Space Theory of Traces and Intermediate Spaces	89
4.1	Introduction,	89
4.2	Hilbert Spaces $H^m(\Omega)$ of Integer Order,	90
4.3	Hilbert Spaces $H^s(\mathbb{R}^n)$ for Real $s \geq 0$,	92
4.4	Duals of Hilbert Spaces,	96
4.5	Duals of Spaces $H^s(\mathbb{R}^n)$ and $H^m(\Omega)$,	104
4.6	The Trace Theorem for $H^m(\mathbb{R}_+^n)$,	112
4.7	Intermediate and Interpolation Spaces,	121
4.8	Interpolation Theory in Hilbert Spaces,	128
4.9	Hilbert Spaces $H^s(\partial\Omega)$,	137
4.10	The Trace Theorem for $H^s(\Omega)$,	141
	References,	143
Chapter 5	Some Elements of Elliptic Theory	145
5.1	Introduction,	145
5.2	Linear Elliptic Operators,	146
5.3	Boundary Conditions,	152
5.4	Green's Formulas,	162
5.5	Regularity Theory in $H^s(\Omega)$, $s \geq 2m$,	169
5.6	Compatibility Conditions—Existence and Uniqueness in $H^s(\Omega)$, $s \geq 2m$,	176
5.7	Existence and Regularity Theory in $H^s(\Omega)$, $s < 2m$,	182
	References,	192

PART II THE THEORY OF FINITE ELEMENTS

Chapter 6	Finite-Element Interpolation	197
6.1	Introduction, 197	
6.2	Connectivity of Finite-Element Models of Domains $\Omega \subset \mathbb{R}^n$, 198	
6.3	Local and Global Representations of Functions, 206	
6.4	Restrictions, Prolongations, and Projections, 215	
6.5	Conjugate Basis Functions, 221	
6.6	Finite-Element Families, 235	
6.7	Accuracy of Finite-Element Interpolations, 264	
	References, 283	
Chapter 7	Variational Boundary-Value Problems	286
7.1	Introduction, 286	
7.2	Formulation of Variational Boundary-Value Problems, 289	
7.3	Coercive Bilinear Forms, 300	
7.4	Weak Coerciveness, 310	
7.5	Existence and Uniqueness of Solutions, 315	
	References, 321	
Chapter 8	Finite-Element Approximations of Elliptic Boundary-Value Problems	323
8.1	Introduction, 323	
8.2	Galerkin Approximations, 323	
8.3	Existence and Uniqueness of Galerkin Approximations, 326	
8.4	Finite-Element Approximations, 330	
8.5	Properties of Finite-Element Subspaces, 334	
8.6	Error Estimates, 342	
8.7	Pointwise and $L_\infty(\Omega)$ Error Estimates, 348	
8.8	Quadrature, Boundary, and Data Errors, 350	

8.9	H^{-1} Finite-Element Methods,	365
8.10	Hybrid and Mixed Finite-Element Methods,	368
	References,	387
Chapter 9	Time-Dependent Problems	390
9.1	Introduction,	390
9.2	Finite-Element Models of the Diffusion Equation,	391
9.3	Semidiscrete L_2 Galerkin Approximations,	393
9.4	Elements of Semigroup Theory,	395
9.5	Semigroup Methods for Galerkin Approxima- tions,	401
9.6	Hyperbolic Equations of Second Order,	409
9.7	First-Order Hyperbolic Equations,	415
	References,	418
Author Index		421
Subject Index		423

