CONTENTS

Preface xi
Note to the Reader xiii

1 Basic Concepts of the Finite Element Method

1.1 Assemblages and Networks 2
1.2 Evolution of the Finite Element Method 17
1.3 A One-Dimensional Example of the Variational Finite Element Method 19
References 40

2 The Variational Finite Element Method

2.1 A Two-Dimensional Heat Flow Example 42
References 71

3 Computer Programming of the Finite Element Method

3.1 Computational Development and the Finite Element Method 72
3.2 A Finite Element Program for the Laplace Problem 73
3.3 Program Modifications 86
References 91

4 Boundary Conditions

4.1 Classification of Boundary Conditions 93
4.2 Boundary Conditions via Line or Surface Integrals 94
4.3 Alternative Formulations for Boundary Conditions 99
References 100

5 Hermitian Elements, Condensation, and Coupled Boundary Conditions

5.1	Problem Statement and Selection of Element	102
5.2	Determination of the Element Matrix Equations	107
5.3	Condensation	113
5.4	Assembly and Insertion of Boundary Conditions	115
5.5	Computer Programming	129
5.6	Extension to Higher Order Elements and More Complex Boundary Conditions	136
	References	137

6 Economization of Core Storage, Partitioning, and Tridiagonalization

6.1	Bandwidth	138
6.2	Storage Modes	143
6.3	Relative Merits of Assembly by Nodes and Elements	145
6.4	Tridiagonalization	146
	References	151

7 The Variational Calculus and Its Application

7.1	Maxima and Minima of Functions	152
7.2	Lagrange Multipliers	155
7.3	Maxima and Minima of Functionals	157
7.4	Admissibility and Finite Elements	162
7.5	Variational Principles in Physical Phenomena	164
	References	165

8 Convergence, Completeness, and Conformity

8.1	Accuracy, Stability, and Convergence in Numerical Computation	167
8.2	Errors in the Finite Element Method	169
8.3	Trial Function Error and Completeness	169
8.4	Trial Function Error and Conformity	171
8.5	Trial Function Error and Nonconformity	173
8.6	Nonconformity, Noncompleteness, and Accuracy	173
8.7	The Patch Test	175
8.8	Admissibility	176
8.9	Physical Equivalents to Completeness and Conformity	177
8.10	Completeness and Geometric Isotropy	178
	References	179

9 Elements and Their Properties

9.1	Element Classification	182
9.2	Element Shape Functions	182
9.3	Natural Coordinates	189
9.4	One-Dimensional Elements	195
9.5	Two-Dimensional Elements	195
9.6	Three-Dimensional Elements	209
9.7	Isoparametric Elements	213
9.8	Transformation from Local to Global Coordinates	215
9.9	Selection of an Element	216
	References	217

10 Equation Solvers and Programming Techniques

10.1	Selection of an Equation Solver	221
10.2	Direct Solution Methods	222
10.3	Iterative Methods	234
10.4	Special Techniques to Aid Equation Solving	243
	References	248

11 Selected Applications of the Finite Element Method

11.1	Solid Mechanics—Plane Strain and Plane Stress	252
11.2	Three-Dimensional Stress Analysis	259
11.3	Acoustics, Electromagnetic Waves, and Seiche Motion	262
11.4	Time-Dependent Problems	265
11.5	Other Applications	266
	References	266

12 Other Finite Element Methods

12.1	Residual Finite Element Methods	268
12.2	Galerkin Finite Element Method	270
12.3	Least-Squares Finite Element Method	276
12.4	Direct Finite Element Method	278
	References	279

Appendix A Matrix Algebra

A.1	Matrix Definitions	281
A.2	Matrix Algebra	284
A.3	Quadratic and Linear Forms	289
	References	290

Appendix B Matrix Calculus

B.1 Differentiation of Matrices 291
B.2 Partial Differentiation of Matrices 292
References 294

Appendix C Determinants

Text 295

Index **297**