

CONTENTS

PREFACE	vii
ACKNOWLEDGMENTS	ix
GREEK ALPHABET	xi
PART 1 BACKGROUND MATERIAL	1
1. INTRODUCTION	3
2. DISCRETE FUNCTION ANALYSIS	12
2.1 Introduction	12
2.2 The Shifting Operators	12
2.3 The Difference Operators	17
2.4 The Factorial Functions	20
2.5 The Binomial Coefficients	26
2.6 Useful Identities	32
2.7 Summation Formulae	36
2.8 Difference Equations	40
Exercises	48
References	53
3. THE DISCRETE ORTHOGONAL POLYNOMIALS	55
3.1 Introduction	55
	xiii

3.2	The Discrete Legendre Polynomials	57
3.3	The Discrete Laguerre Polynomials	63
	Exercises	67
	References	71
4.	STATE-VECTORS AND TRANSITION MATRICES	72
4.1	Introduction	72
4.2	The Derivative Polynomial Model	73
4.3	The Difference Vectors	79
4.4	The Link Matrices	81
4.5	The Lagrange Estimator	87
4.6	Constant-Coefficient Linear Differential Equations	92
4.7	Time-Varying Linear Differential Equations	99
4.8	Nonlinear Systems	105
	Notes	116
	Exercises	116
	References	123
5.	STATISTICAL BACKGROUND	124
5.1	Introduction	124
5.2	Concepts from Probability Theory	125
5.3	Properties of Covariance Matrices	134
5.4	The Covariance Matrix of the Input Errors	138
5.5	The Covariance Matrix of the Output Errors	142
5.6	The Gaussian and Chi-Squared Density Functions	153
	Exercises	158
	References	164
6.	EXACTNESS, LEAST-SQUARES AND MINIMUM-VARIANCE	165
6.1	Introduction	165
6.2	The Observation Scheme	166
6.3	The Exactness Constraint	171
6.4	Further Relationships	178
6.5	Least-Squares Estimation	180
6.6	Minimum-Variance Estimation	185
6.7	General Aspects of Minimum-Variance	193
6.8	Uniqueness of Least-Squares and Minimum-Variance	198
6.9	Weighted Least-Squares	201
6.10	The Residuals	203
	Notes	208
	Exercises	209
	References	219

PART 2 FIXED-MEMORY FILTERING	221
7. THE FIXED-MEMORY POLYNOMIAL FILTER	223
7.1 Introduction	223
7.2 Classical Least-Squares	225
7.3 The Orthogonal Polynomial Approach	228
7.4 The General Form	235
7.5 Variance Reduction	240
7.6 Dependence of the Covariance Matrix on L and τ	246
7.7 Dependence of the Covariance Matrix on m and h	251
7.8 The Variance Reduction Factors	256
7.9 A Simple Example	260
7.10 The Systematic Errors	264
7.11 Behavior of the Systematic Errors with h and m	268
7.12 Behavior of the Systematic Errors with L and τ	271
7.13 Balancing the Systematic and Random Errors	275
7.14 Trend Removal	277
7.15 Cascaded Simple Averaging	281
Exercises	281
References	290
8. GENERALIZED FIXED-MEMORY FILTERING	291
8.1 Introduction	291
8.2 The Basic Scheme	293
8.3 Generalized Fixed-Memory Polynomial Filters	295
8.4 Nonlinear Systems	298
8.5 Obtaining a Nominal Trajectory	300
8.6 Iterative Differential-Correction	303
8.7 Iteration Procedure	311
8.8 Comments on Computational Problems	314
8.9 Relationship between M and Φ	320
Exercises	327
References	338
PART 3 EXPANDING-MEMORY FILTERING	339
9. THE EXPANDING-MEMORY POLYNOMIAL FILTER	341
9.1 Introduction	341
9.2 The Approximating Polynomial	342

9.3	Recursive Formulation	348
9.4	Coupling Procedure	355
9.5	Stability	362
9.6	Initialization	367
9.7	Variance Reduction	369
9.8	Systematic Errors	372
	Exercises	372
	References	376
10. GENERALIZED EXPANDING-MEMORY FILTERS— THE BAYES FORMULATION		377
10.1	Introduction	377
10.2	The Bayes Filter—Linear Case	379
10.3	<i>The A Priori</i> Estimate	389
10.4	Basic Structure of the Bayes Filter	393
10.5	Properties of the Covariance Matrix	398
10.6	Computational Aspects	407
	Exercises	411
	References	423
11. BAYES ALGORITHM WITH ITERATIVE DIFFERENTIAL-CORRECTION		424
11.1	Introduction	424
11.2	Iterative Differential-Correction—Computational Procedure	428
11.3	Control of Error-Propagation	434
11.4	Convergence of Iterative Differential-Correction Notes	436
	Exercises	442
	References	443
		460
12. GENERALIZED EXPANDING-MEMORY FILTERS— THE KALMAN FORMULATION		461
12.1	Introduction	461
12.2	The Kalman Filter—Linear Case with No Driving-Noise	463
12.3	Criteria Satisfied by the Bayes and Kalman Filters	466
12.4	Properties of the Kalman Filter without Driving-Noise	470
12.5	Iterative Differential-Correction Notes	479
	Exercises	482
	References	493

PART 4 FADING-MEMORY FILTERING	495
13. THE FADING-MEMORY POLYNOMIAL FILTER	497
13.1 Introduction	497
13.2 Discounted Least-Squares	498
13.3 Recursive Formulation	504
13.4 Stability	507
13.5 Coupling Procedure	510
13.6 Variance Reduction	519
13.7 Comparison with the Fixed-Memory Polynomial Filters	532
13.8 Initialization	536
13.9 Systematic Errors	538
Exercises	547
References	553
14. GENERALIZED FADING-MEMORY FILTERS	555
14.1 Introduction	555
14.2 Exponential Stress	556
14.3 Minimization Procedure	560
14.4 Recursive Formulation	563
14.5 Properties of the Generalized Fading-Memory Filters	570
14.6 Systematic Error Control	573
14.7 Stability Analysis	576
14.8 Detection of Systematic Errors	594
14.9 Iterative Differential-Correction	596
Exercises	597
References	602
15. THE KALMAN FILTER WITH DRIVING-NOISE	603
15.1 Introduction	603
15.2 Random Forcing-Functions	604
15.3 Estimating in the Presence of Driving-Noise	609
15.4 The Kalman Filter with and without Driving-Noise	614
15.5 Practical Aspects	616
Exercises	618
References	621
APPENDIX I PROPERTIES OF THE DISCRETE LEGENDRE POLYNOMIALS	623
APPENDIX II PROOF OF EQUATION (13.3.8)	628
INDEX	633