

CONTENTS

CONTENTS	V
FOREWORD TO THE ENGLISH TRANSLATION	X
TRANSLATOR'S PREFACE	XI
FOREWORD	XII
INTRODUCTION	XVI

CHAPTER I

A classification of systems of elements in a Hilbert space

1. Minimal systems.	1
2. Strongly minimal and almost orthonormal systems.	4
3. Similar and semi-similar operators.	7
4. Comparison theorems	8
5. Properties of best approximation.	13

CHAPTER II

The stability of the Ritz and Bubnov-Galerkin processes for stationary problems

6. Discussion of the Ritz-process	16
7. Limit properties of the Ritz-coefficients	22
8. Examples introducing the concept of numerical stability	32
9. On the stability of the Ritz-process	41
10. The stability of approximate solutions.	47
11. The condition number of the Ritz-matrix.	51
12. The solution of the Ritz-process by iteration	53
13. Extension of the concept of stability.	57

14. The stability of the Bubnov-Galerkin-process for stationary problems.	63
15. Remarks about the use of non-strongly minimal systems	70
16. Another concept of stability	76

CHAPTER III

On the stability of the Bubnov-Galerkin-Process for non-stationary problems

17. The Bubnov-Galerkin-process for non-stationary problems	82
18. Equations of parabolic type	88
19. A more general equation of the first order	96
20. The S. L. Sobolev equations	101
21. Equations of hyperbolic type.	105

CHAPTER IV

The residual of an approximate solution

22. A residual theorem	109
23. Non-degenerate ordinary differential operators of the second order	110
24. Degenerate ordinary differential operators of the second order	114
25. Ordinary differential operators with order higher than the second.	120
26. Elliptic operators of the second order	123
27. Another approach for studying the residual.	126
28. Polynomial coordinate systems	129

CHAPTER V

On the rational choice of coordinate functions

29. General remarks	132
30. Ordinary differential equations of the second order . . .	137
31. Degenerate equations	145
32. Ordinary differential equations of the fourth order . . .	150

33. Two-dimensional elliptic equations: the first boundary value problem	152
34. Two-dimensional elliptic equations with natural boundary conditions	157
35. Three-dimensional problems	159
36. Systems of ordinary differential equations	164
37. Systems of partial differential equations	167
38. Coordinate systems for the method of least squares . . .	170
39. Integral equations.	177

CHAPTER VI

Infinite regions and other singular problems

40. Preliminary remarks.	187
41. Second order elliptic equations in an infinite region . . .	190
42. The divergence condition	195
43. Other conditions for solvability.	200
44. Homogeneous differential equations.	202
45. Degenerate equations in a finite region.	205
46. Coordinate systems for homogeneous problems defined on an infinite interval.	210
47. Coordinate systems for multi-dimensional problems defined on infinite regions with finite boundaries	218
48. Coordinate systems for regions with infinite boundaries	223
49. Examples	227
50. Coordinate systems for degenerate equations defined in finite regions	231

CHAPTER VII

The stability of the Ritz-process for eigenvalue problems

51. The general theorem.	235
52. The stability of the Ritz-process for eigenvalue problems	240
53. The stability of the Ritz-process for the eigenfunction problem	242

CHAPTER VIII

The effect of error in the equation

54. The formulation of the problem and an estimate for the error in the solution.	248
55. Application to second order equations.	251
56. Application to the linear theory of shells. Formulation of the problem	255
57. The potential energy of deformation of a shell	256
58. The shell-operator.	261
59. Shells similar to plane plates	264
60. The purely rotational state of stress	270
61. Helical shells.	271
62. A numerical example	278

CHAPTER IX

Variational methods for non-linear problems

63. Preliminary remarks and auxiliary information	283
64. Positive operators in a Banach space	288
65. Some theorems of the calculus of variations	289
66. The existence of a solution of a variational problem.	292
67. The energy space of a non-linear problem	298
68. Functionals in the theory of plasticity and their extension	300
69. Functionals in the theory of plasticity and their extension (continuation)	307

CHAPTER X

The numerical solution of non-linear variational problems

70. The Ritz-process and Bubnov-Galerkin-process	317
71. Application of the Newton-Kantorovich-method	321
72. Differentiation with respect to a parameter.	325
73. The application to finite difference equations	332
74. An example	344
75. The Kachanov-method	354

76. The stability of the Ritz-process for non-linear problems	356
BIBLIOGRAPHY	362
INDEX	370