

CONTENTS

Preface	vii
Acknowledgements	x
Chapter 1—Introduction	1
1.1 Objects and variables	1
1.2 Some multivariate problems and techniques	2
1.3 The data matrix	8
1.4 Summary statistics	9
1.5 Linear combinations	13
1.6 Geometrical ideas	16
1.7 Graphical representations	17
*1.8 Measures of multivariate skewness and kurtosis	20
Exercises and complements	22
Chapter 2—Basic Properties of Random Vectors	26
2.1 Cumulative distribution functions and probability density functions	26
2.2 Population moments	28
2.3 Characteristic functions	33
2.4 Transformations	35
2.5 The multinormal distribution	36
2.6 Some multivariate generalizations of univariate distributions	43
2.7 Families of distributions	45
2.8 Random samples	49
2.9 Limit theorems	51
Exercises and complements	53
Chapter 3—Normal Distribution Theory	59
3.1 Characterization and properties	59
3.2 Linear forms	62
3.3 Transformations of normal data matrices	64
3.4 The Wishart distribution	66

3.5	The Hotelling T^2 distribution	73
3.6	Mahalanobis distance	76
3.7	Statistics based on the Wishart distribution	80
3.8	Other distributions related to the multinormal	85
	Exercises and complements	86
Chapter 4—Estimation		96
4.1	Likelihood and sufficiency	96
4.2	Maximum likelihood estimation	102
4.3	Other techniques and concepts	109
	Exercises and complements	113
Chapter 5—Hypothesis Testing		120
5.1	Introduction	120
5.2	The techniques introduced	123
5.3	The techniques further illustrated	131
*5.4	The Behrens–Fisher problem	142
5.5	Simultaneous confidence intervals	144
5.6	Multivariate hypothesis testing: some general points	147
*5.7	Non-normal data	148
5.8	A non-parametric test for the bivariate two-sample problem	149
	Exercises and complements	151
Chapter 6—Multivariate Regression Analysis		157
6.1	Introduction	157
6.2	Maximum likelihood estimation	158
6.3	The general linear hypothesis	161
6.4	Design matrices of degenerate rank	164
6.5	Multiple correlation	167
6.6	Least squares estimation	171
6.7	Discarding of variables	175
	Exercises and complements	180
Chapter 7—Econometrics		185
7.1	Introduction	185
7.2	Instrumental variables and two-stage least squares	186
7.3	Simultaneous equation systems	191
7.4	Single-equation estimators	199
7.5	System estimators	203
7.6	Comparison of estimators	208
	Exercises and complements	208
Chapter 8—Principal Component Analysis		213
8.1	Introduction	213
8.2	Definition and properties of principal components	214
8.3	Sampling properties of principal components	229
8.4	Testing hypotheses about principal components	233
8.5	Correspondence analysis	237
8.6	Allometry—the measurement of size and shape	239

8.7	Discarding of variables	242
8.8	Principal component analysis in regression	244
	Exercises and complements	246
Chapter 9	—Factor Analysis	255
9.1	Introduction	255
9.2	The factor model	256
9.3	Principal factor analysis	261
9.4	Maximum likelihood factor analysis	263
9.5	Goodness of fit test	267
9.6	Rotation of factors	268
9.7	Factor scores	273
9.8	Relationships between factor analysis and principal component analysis	275
9.9	Analysis of covariance structures	276
	Exercises and complements	276
Chapter 10	—Canonical Correlation Analysis	281
10.1	Introduction	281
10.2	Mathematical development	282
10.3	Qualitative data and dummy variables	290
10.4	Qualitative and quantitative data	293
	Exercises and complements	295
Chapter 11	—Discriminant Analysis	300
11.1	Introduction	300
11.2	Discrimination when the populations are known	301
11.3	Discrimination under estimation	309
11.4	Is discrimination worthwhile?	318
11.5	Fisher's linear discriminant function	318
11.6	Probabilities of misclassification	320
11.7	Discarding of variables	322
11.8	When does correlation improve discrimination?	324
	Exercises and complements	325
Chapter 12	—Multivariate Analysis of Variance	333
12.1	Introduction	333
12.2	Formulation of multivariate one-way classification	333
12.3	The likelihood ratio principle	334
12.4	Testing fixed contrasts	337
12.5	Canonical variables and a test of dimensionality	338
12.6	The union intersection approach	348
12.7	Two-way classification	350
	Exercises and complements	356
Chapter 13	—Cluster Analysis	360
13.1	Introduction	360
13.2	A probabilistic formulation	361
13.3	Hierarchical methods	369
13.4	Distances and similarities	375

13.5 Other methods and comparative approach	384
Exercises and complements	386
Chapter 14—Multidimensional Scaling	394
14.1 Introduction	394
14.2 Classical solution	397
14.3 Duality between principal coordinate analysis and principal component analysis	404
14.4 Optimal properties of the classical solution and goodness of fit . .	406
14.5 Seriation	409
14.6 Non-metric methods	413
14.7 Goodness of fit measure: Procrustes rotation	416
*14.8 Multi-sample problem and canonical variates	419
Exercises and complements	420
Chapter 15—Directional Data	424
15.1 Introduction	424
15.2 Descriptive measures	426
15.3 Basic distributions	428
15.4 Distribution theory	435
15.5 Maximum likelihood estimators for the von Mises-Fisher distribution	437
15.6 Test of uniformity: the Rayleigh test	439
15.7 Some other problems	441
Exercises and complements	446
Appendix A—Matrix Algebra	452
A.1 Introduction	452
A.2 Matrix operations	455
A.3 Further particular matrices and types of matrix	460
A.4 Vector spaces, rank, and linear equations	462
A.5 Linear transformations	465
A.6 Eigenvalues and eigenvectors	466
A.7 Quadratic forms and definiteness	474
*A.8 Generalized inverse	476
A.9 Matrix differentiation and maximization problems	478
A.10 Geometrical ideas	481
Appendix B—Univariate Statistics	486
B.1 Introduction	486
B.2 Normal distribution	486
B.3 Chi-squared distribution	487
B.4 <i>F</i> and beta variables	487
B.5 <i>t</i> distribution	488
Appendix C—Tables	489
Table C. 1 Upper percentage points of the χ^2_v distribution	490

Table C.2 Upper percentage points of the t_ν distribution 491

Table C.3 Upper percentage points of the F_{ν_1, ν_2} distribution 492

Table C.4 Upper percentage points θ_α of $\theta(p, \nu_1, \nu_2)$, the largest
 eigenvalue of $|\mathbf{B} - \theta(\mathbf{W} + \mathbf{B})| = 0$ for $p = 2$ 494

References **497**

List of Main Notations and Abbreviations 508

Subject Index 510

Author Index 519