

Contents

Editor's Statement	xiii
Preface	xv
Introduction	xix
Chapter 1 Basic Definitions and Properties	1
1.1 Introduction	1
1.2 Basic Definitions	2
1.3 The Algebraic Case	4
1.4 Pólya and Birkhoff Matrices	7
1.5 Regularity Theorems	9
1.6 The Interpolating Polynomial	12
Chapter 2 Further Elementary Theorems	14
2.1 Estimation of the Defect; Theorem of Budan–Fourier	14
2.2 Count of Pólya and Birkhoff Matrices	17
2.3 Symmetry	20
2.4 Trigonometric Interpolation	22
2.5 Notes	24
Chapter 3 Coalescence of Rows	26
3.1 Introduction	26
3.2 Level Functions and Coalescence	27

3.3	Coalescence in a Matrix	29
3.4	Shifts and Differentiation of Determinants	32
3.5	Permutation Numbers and Taylor Expansions	35
Chapter 4	Applications of Coalescence	38
4.1	Conditionally Regular and Strongly Singular Matrices	38
4.2	The Function $\delta(E)$	41
4.3	The Theorem of Ferguson	43
4.4	Singularity and Permutation Numbers	45
4.5	Applications	47
4.6	The Coalescence Constant; Signs of Determinants	49
Chapter 5	Rolle Extensions and Independent Sets of Knots	52
5.1	Rolle Extensions	52
5.2	An Auxiliary Theorem	55
5.3	Markov's Inequality and Applications	56
5.4	Independent Sets of Knots	58
5.5	Applications; Birkhoff Systems	61
5.6	Continuity of Interpolation	63
Chapter 6	Singular Matrices	65
6.1	Introduction; Supported Singletons	65
6.2	Proof of Theorem 6.2	67
6.3	Almost-Simple Matrices	70
6.4	Probability of Regularity	72
6.5	Notes	78
Chapter 7	Zeros of Birkhoff Splines	79
7.1	Birkhoff's Kernel	79
7.2	Birkhoff's Identity	81
7.3	Splines; Diagrams, Boundary	85
7.4	Count of Zeros of Splines	88
7.5	Corollaries; Birkhoff–Lorentz Theorem	91
7.6	Applications to Birkhoff Interpolation	94
7.7	Notes	99
Chapter 8	Almost-Hermitian Matrices; Special Three-Row Matrices	100
8.1	Introduction	100
8.2	Almost-Hermitian Matrices	101
8.3	Reciprocal Almost-Hermitian Matrices	105
8.4	Null Curves for the Matrices $E(p, q; k_1, k_2)$	107

8.5	The Exterior Symmetric Case	113
8.6	The Interior Case	118
8.7	Notes	120
Chapter 9	Applications	121
9.1	Introduction	121
9.2	Monotone Approximation	122
9.3	Approximation with Restrictions for Derivatives	125
9.4	The Uniqueness Theorem	127
9.5	Simultaneous Approximation	129
9.6	Chebyshev Systems on $[-1, +1]$	131
9.7	Notes	132
Chapter 10	Birkhoff Quadrature Formulas	135
10.1	Definitions; q -Regular Pairs E, X	135
10.2	q -Regular Matrices	140
10.3	Pólya Conditions; Gaussian Formulas	141
10.4	Kernels of Quadrature Formulas	145
10.5	Dyn's Theorems	146
10.6	Notes	150
Chapter 11	Interpolation at the Roots of Unity	152
11.1	Lacunary Interpolation	152
11.2	Regularity of (m_0, m_1, \dots, m_q) Interpolation	153
11.3	Estimates on the Fundamental Polynomials	157
11.4	Convergence of Interpolation at the Roots of Unity	161
11.5	Trigonometric Interpolation	164
11.6	Notes	168
Chapter 12	Turán's Problem of $(0, 2)$ Interpolation	170
12.1	Regularity of Sets of Knots	170
12.2	Further Properties of Legendre Polynomials	173
12.3	Fundamental Polynomials ρ_i	176
12.4	Fundamental Polynomials r_i	180
12.5	Properties of the Interpolation Operator; Convergence	183
12.6	Notes	188
Chapter 13	Birkhoff Interpolation by Splines	190
13.1	Introduction	190
13.2	Basic Definitions	192
13.3	Duality	193

13.4	Pólya Conditions for Spline Interpolation	196
13.5	Decomposition of Matrices	199
13.6	Notes	202
Chapter 14	Regularity Theorems and Self-Dual Problems	203
14.1	Historical Remarks	203
14.2	Estimation of $ F_\phi^1 $	205
14.3	Goodman's Theorem	208
14.4	Regularity of Self-Dual Matrices	210
14.5	Best Quadrature Formulas	212
14.6	Interpolation with Derivative of Minimal Norm	215
14.7	Notes	219
	Bibliography and References	220
	Symbol Index	233
	Subject Index	235

