

CONTENTS

PREFACE	v
CHAPTER 1. INTRODUCTION	1
1.1. Numerical analysis	1
1.2. Approximation	2
1.3. Errors	4
1.4. Significant figures	8
1.5. Error bounds	14
1.6. Random errors	17
1.7. Mathematical preliminaries	21
1.8. Supplementary references	26
Problems	26
CHAPTER 2. INTERPOLATION WITH DIVIDED DIFFERENCES	35
2.1. Introduction	35
2.2. Linear interpolation	35
2.3. Divided differences	38
2.4. Second-order interpolation	41
2.5. Newton's fundamental formula	43
2.6. Error formulas	45
2.7. Iterated interpolation	49
2.8. Inverse interpolation	50
2.9. Supplementary references	53
Problems	53
CHAPTER 3. LAGRANGIAN METHODS	60
3.1. Introduction	60
3.2. Lagrange's interpolation formula	60
3.3. Numerical differentiation and integration	64
3.4. Calculation	68
3.5. Newton-Cotes integration formulas	71
3.6. Use of integration formulas	76
3.7. Asymptotic behavior of Newton-Cotes formulas	79
3.8. Differentiation formulas	82
3.9. Supplementary references	84
Problems	84
CHAPTER 4. FINITE-DIFFERENCE INTERPOLATION	91
4.1. Introduction	91
4.2. Difference notations	91

4.3. Newton forward- and backward-difference formulas	94
4.4. Gaussian formulas	97
4.5. Stirling's formula	99
4.6. Bessel's formula	101
4.7. Everett's formulas	103
4.8. Use of interpolation formulas	105
4.9. Propagation of inherent errors	110
4.10. Throwback techniques	112
4.11. Interpolation series	114
4.12. Tables of interpolation coefficients	118
4.13. Supplementary references	121
Problems	121
CHAPTER 5. OPERATIONS WITH FINITE DIFFERENCES	128
5.1. Introduction	128
5.2. Difference operators	128
5.3. Differentiation formulas	134
5.4. Newtonian integration formulas	138
5.5. Newtonian formulas for repeated integration	142
5.6. Central-difference integration formulas	144
5.7. Subtabulation	146
5.8. Summation. The Euler-Maclaurin sum formula	149
5.9. Formulas of Gregory and Gauss. Euler's transformation	155
5.10. Special integration formulas	160
5.11. Error terms in integration formulas	161
5.12. Other representations of error formulas	170
5.13. Supplementary references	174
Problems	175
CHAPTER 6. NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS	188
6.1. Introduction	188
6.2. Formulas of open type	188
6.3. Formulas of closed type	191
6.4. Start of solution	192
6.5. Methods based on open-type formulas	197
6.6. Methods based on closed-type formulas	199
6.7. The special case $F = Ay$	202
6.8. Propagated-error bounds	208
6.9. Convergence of iterations	212
6.10. Application to equations of higher order	214
6.11. Propagated-error bounds	219
6.12. Special second-order equations	223
6.13. Change of interval	227
6.14. Use of higher derivatives	229
6.15. A simple Runge-Kutta method	233
6.16. Runge-Kutta methods of higher order	236
6.17. Boundary-value problems	239
6.18. Linear characteristic-value problems	243
6.19. Selection of a method	247
6.20. Supplementary references	249
Problems	249

CHAPTER 7. LEAST-SQUARES POLYNOMIAL APPROXIMATION	258
7.1. Introduction	258
7.2. The principle of least squares	258
7.3. Least-squares approximation over discrete ranges	261
7.4. Orthogonal polynomials	269
7.5. Legendre approximation	272
7.6. Laguerre approximation	274
7.7. Hermite approximation	277
7.8. Chebyshev approximation	279
7.9. Factorial power functions and summation formulas	282
7.10. Polynomials orthogonal over discrete ranges	287
7.11. Gram approximation	288
7.12. Example. Five-point least-squares approximation	291
7.13. Smoothing formulas	295
7.14. Supplementary references	302
Problems	302
CHAPTER 8. GAUSSIAN QUADRATURE AND RELATED TOPICS	312
8.1. Introduction	312
8.2. Hermite interpolation	314
8.3. Hermite quadrature	317
8.4. Gaussian quadrature	319
8.5. Legendre-Gauss quadrature	323
8.6. Laguerre-Gauss quadrature	325
8.7. Hermite-Gauss quadrature	327
8.8. Chebyshev-Gauss quadrature	330
8.9. Jacobi-Gauss quadrature	331
8.10. Formulas with assigned abscissas	334
8.11. Radau quadrature	338
8.12. Lobatto quadrature	342
8.13. Chebyshev quadrature	345
8.14. Algebraic derivations	351
8.15. Supplementary references	357
Problems	358
CHAPTER 9. APPROXIMATIONS OF VARIOUS TYPES	368
9.1. Introduction	368
9.2. Fourier approximation: continuous range	369
9.3. Fourier approximation: discrete range	373
9.4. Exponential approximation	378
9.5. Determination of constituent periodicities	382
9.6. Optimum polynomial interpolation with selected abscissas	386
9.7. Chebyshev interpolation	389
9.9. Economization of power series	391
9.9. Approximation by continued fractions	395
9.10. Nature of continued-fraction approximations	399
9.11. Determination of convergents of continued fractions	402
9.12. Thiele's continued-fraction expansions	406
9.13. Supplementary references	412
Problems	412

CHAPTER 10. NUMERICAL SOLUTION OF EQUATIONS	424
10.1. Introduction	424
10.2. Sets of linear equations	424
10.3. The Gauss reduction	428
10.4. The Crout reduction	429
10.5. Determination of the inverse matrix	434
10.6. Inherent errors	436
10.7. Gauss-Seidel iteration and relaxation	439
10.8. Iterative methods for solving nonlinear equations	443
10.9. Iterated synthetic division	451
10.10. Bernoulli's iteration	458
10.11. Graeffe's root-squaring technique	462
10.12. Iterated synthetic division with quadratic factors. Lin iteration	468
10.13. Bairstow iteration	472
10.14. Supplementary references	476
Problems	477
APPENDIX A. JUSTIFICATION OF THE CROUT REDUCTION	486
APPENDIX B. BIBLIOGRAPHY	490
APPENDIX C. DIRECTORY OF METHODS	502
INDEX	507

