Contents

Symbols, **xiii–xiv**

PART ONE GRADIENT OPTIMIZATION

Chapter 1 Introduction **3**

1.1 Statement of the General Problem, 3

Chapter 2 Basic Mathematical Concepts **7**

2.1 Some Fundamental Definitions, 7
2.2 Linear and Bilinear Operators, 12
2.3 Tangency and the Derivative of an Operator, 18
2.4 The Derivative of a Composite Operator and the Second Derivative of an Operator, 23
2.5 The Taylor Series, Expansion of a Functional, 28
2.6 The Minimum of a Functional, 30
Problems, 36
References, 38

Chapter 3 Conjugate Gradient Descent **39**

3.1 Introduction, 39
3.2 Conjugate Descent, 41
3.3 Conjugate Gradient Descent of a Quadratic Functional, 44
3.4 Conjugate Gradient Descent of a Smooth Functional, 56

3.5 Scaled Conjugate Gradient Descent in $\mathscr{R}^n$, 63
Problems, 83
References, 84

PART TWO NONLINEAR CONTROL

Chapter 4 The Gradient of the Cost Functional for Some Common Cases of Interest in Control Systems **89**

4.1 A General Method for Finding the Gradient of a Functional, 89
4.2 The Cost Functional of Interest in Control System Design, 92
4.3 The Gradient for the Case of a Continuous Function Input, 94
4.4 The Gradient in the Space of Input Control Parameters, 103
4.5 The Case of a Sampled Input, 111
4.6 Summary 115
Problems, 117
References, 119

Chapter 5 The Gradient of the Cost Functional in Some Special Cases of Interest in Control Systems **120**

5.1 Some Motivating Considerations, 120
5.2 Piecewise Continuous Control Inputs, 121
5.3 The Gradient of a Cost Functional on the Space of Pulse-Width Modulated Control Inputs, 139
5.4 Variable Initial Conditions—Periodic Boundary Conditions, 148
5.5 Systems with Delay, 155

Chapter 6 Design of Controllers by Gradient Methods **173**

6.1 The Basic Approach, 173
6.2 State Variable Feedback Controller, 174
6.3 The Model Follower Controller, 185
6.4 Design of Controllers Using Stochastic Test Signals, 194
6.5 Cascade-Compensated Feedback Tracking System, 210
6.6 Controller Design and Parameter Identification for a Stirred Tank Mixer, 219
References 232

Appendices 233

A Properties of a Positive-Definite Linear Operator 233
B Some Practical Aspects of the Problem of Finding α to Minimize $F(x_i + \alpha p_i)$ in the Application of Conjugate Descent 236
C Solution to the Differential Equation $\dot{x} = A(t)x + B(t)u$ 240
D The Adjoint System 242
E A State Transition Matrix Solution to a System of Linear, Time-Varying, Difference-Differential Equations 244
F Solution to the Discrete, Linear System $x_{k+1} = A_k x_k + B_k u_k$ 248
G A Program for Conjugate Gradient Descent 250

Answers to Selected Problems 255

Index 261