



# Table of Contents

|                                                             |    |
|-------------------------------------------------------------|----|
| 1. Binomial Identities . . . . .                            | 5  |
| 1.1 Summary of Useful Identities . . . . .                  | 5  |
| 1.2 Deriving the Identities . . . . .                       | 7  |
| 1.3 Inverse Relations . . . . .                             | 9  |
| 1.4 Operator Calculus . . . . .                             | 12 |
| 1.5 Identities with the Harmonic Numbers . . . . .          | 14 |
| 2. Recurrence Relations . . . . .                           | 15 |
| 2.1 Linear Recurrence Relations . . . . .                   | 15 |
| 2.1.1 Finite History . . . . .                              | 16 |
| 2.1.1.1 Constant Coefficients . . . . .                     | 16 |
| 2.1.1.2 Variable Coefficients . . . . .                     | 18 |
| 2.1.2 Full History . . . . .                                | 21 |
| 2.1.2.1 Differencing . . . . .                              | 21 |
| 2.1.2.2 By Repertoire . . . . .                             | 21 |
| 2.2 Nonlinear Recurrence Relations . . . . .                | 25 |
| 2.2.1 Relations with Maximum or Minimum Functions . . . . . | 25 |
| 2.2.2 Continued Fractions . . . . .                         | 29 |
| 2.2.3 Doubly Exponential Sequences . . . . .                | 31 |
| 3. Operator Methods . . . . .                               | 35 |
| 3.1 The Cookie Monster . . . . .                            | 35 |
| 3.2 Coalesced Hashing . . . . .                             | 38 |
| 3.3 Open Addressing: Uniform Hashing . . . . .              | 42 |
| 3.4 Open Addressing: Secondary Clustering . . . . .         | 43 |
| 4. Asymptotic Analysis . . . . .                            | 46 |
| 4.1 Basic Concepts . . . . .                                | 46 |
| 4.1.1 Notation . . . . .                                    | 47 |
| 4.1.2 Bootstrapping . . . . .                               | 47 |

|                                                         |     |
|---------------------------------------------------------|-----|
| 4.1.3 Dissecting . . . . .                              | 48  |
| 4.1.4 Limits of Limits . . . . .                        | 49  |
| 4.1.5 Summary of Useful Asymptotic Expansions . . . . . | 51  |
| 4.1.6 An Example . . . . .                              | 52  |
| 4.2 Stieltjes Integration . . . . .                     | 59  |
| 4.2.1 $O$ -notation and Integrals . . . . .             | 61  |
| 4.2.2 Euler's Summation Formula . . . . .               | 62  |
| 4.2.3 A Number Theory Example . . . . .                 | 64  |
| 4.3 Asymptotics from Generating Functions . . . . .     | 69  |
| 4.3.1 Darboux's Method . . . . .                        | 69  |
| 4.3.2 Residue Calculus . . . . .                        | 72  |
| 4.3.3 The Saddle Point Method . . . . .                 | 74  |
| 5. Bibliography . . . . .                               | 81  |
| 6. Appendices . . . . .                                 | 85  |
| A. Lectures . . . . .                                   | 85  |
| B. Homework Assignments . . . . .                       | 87  |
| C. Midterm Exam and Solutions . . . . .                 | 88  |
| D. Final Exam and Solutions . . . . .                   | 99  |
| 7. Index . . . . .                                      | 105 |

