Contents

PREFACE	
INTRODUCTION	1
NOTATION	14
CHAPTER 1. THE FREE BOUNDARY IN THE DIRICHLET PROBLEM FOR SECOND ORDER ELLIPTIC QUASILINEAR EQUATIONS	16
1.1. On the existence of the free boundary	17
1.1a.Onedimensional and radially symmetric solutions	22
1.1b.Interior estimates. Local super and subsolutions	33
1.1c.Boundary estimates. Nondiffusion of the support 1.1d.Solutions with compact support.Global super and	42
subsolutions	48
1.2. Nonexistence of the free boundary. Positivity of solutions 1.2a. On the balance diffusion-absorption. A strong maxi-	54
mum principle	54
1.2b. Criteria on the balance between the data and the	
domain. Gradient estimates	57
1.3. Some applications of the symmetric rearrangement of a function	63
1.3a. A general result. An isoperimetric inequality for the	

1.3b. On the symmetry of the solution and of its null set

1.3c. The free boundary for equations with a general non-

1.4a. On the behaviour of solutions near the free boundary

1.4b. Lebesgue and Hausdorff measure of the free boundary.

Application to domains of boundary having nonnegative

1.4. Further results on the free boundary for semilinear equa-

67

83

86

90

90

96

null set

tions

linear diffusion term

mean curvature

	1.4c.	Regularity of the free boundary and dependence with	
		respect to a parameter in the equation	104
	1.4d.	Geometrical properties of the free boundary	108
1.5.	Biblic	ographical notes	113
CHAPTER		FREE BOUNDARY IN OTHER SECOND ORDER NON LINEAR	440
	PRC	DBLEMS	118
2.1.	Equati	ons with nonmonotone perturbation term	119
	2.1a.	A nonmonotone semilinear equation in exothermical	
		chemical reactions	120
		A nonlinear system	124
	2.1c.	Equilibrium solutions of a degenerate parabolic	105
	0 1 .	equation in biological population models	125
	2.1d.	Nonnegative radial solutions of a nonmonotone semilinear equation in $\ensuremath{\mathbb{R}}^N$	131
2.2.		cional Inequalities and multivalued equations	137
	2.2a.	Existence and location of the free boundary	1 // 1
	0.01	Solutions with compact support	141 151
		Rearrangement and multivalued equations Further results	157
	2.20.	Further results	
2.3.	`	gular equation	161
		On the variational and other limiting solutions	161
	2.3b.	On the existence of the free boundary	168
2.4.	Nonis	otropic equations	177
	2.4a.	Equations in divergenece form. On the diffusion-	
		convection balance	179
	2.4b.	Fully nonlinear equations. Optimal strategy for	104
		the Hamilton-Jacobi-Bellman equation	184
2.5.	Other	boundary-value problems	196
	2.5a.	Nonlinear equations with other boundary conditions	197
	2.5b.	The Signorini problem	201
2.6.	Bibli	ographical notes	207

CHAPTER	3. EXISTENCE AND LOCATION OF THE FREE BOUNDARY BY MEANS OF ENERGY METHODS	212
3.1.	Second order quasilinear equations	212
	3.1a. The main result	212
	3.1b. Proof of the interpolation-trace Lemma	223
3.2.	Quasilinear elliptic equations of arbitrary order	226
3.3.	Bibliographical notes	235
CHARTER	4. THE GENERAL THEORY FOR SECOND ORDER NONLINEAR ELLIPTIC	
CHAPIER	EQUATIONS: A PARTICULAR OVERVIEW	237
4.1.	Solutions in the energy space	238
	4.1a. Some first existence results via minimization of	
	functionals	238
	4.1b. Some extensions of variational problems: Monotone	
	operators and their generalizations	248
	4.1c. On the regularity of solutions. L_{loc}^{∞} -estimates	257
	4.1d. Uniqueness and comparison results. Existence via	
	comparison	263
4.2.	Solutions outside the energy space	273
	4.2a. Semilinear equations in $L^1(\Omega)$ and other spaces	275
	4.2b. Abstract results. Accretive operators. Application	
	to quasilinear equations	284
4.3.	Bibliographical notes	290
REFEREN	CES	292
INDEX		320

INDEX