

CONTENTS

Preface

xvii

Chapter I **Basic Concepts of Control Theory**

1.1. Introduction	1
1.2. Systems and State Variables	2
1.3. Discussion	2
1.4. Control Variables	3
1.5. Criterion Function	4
1.6. Control Processes	5
1.7. Analytic Aspects	6
1.8. Computational Aspects	7
1.9. Event versus Time Orientation	7
1.10. Discrete versus Continuous Bibliography and Comments	8

Chapter 2 **Discrete Control Processes and Dynamic Programming**

2.1. Introduction	10
2.2. Existence of a Minimum	11
2.3. Uniqueness	12
2.4. Dynamic Programming Approach	12
2.5. Recurrence Relations	13
2.6. Imbedding	14
2.7. Policies	15
2.8. Principle of Optimality	16
2.9. Discussion	16
2.10. Time Dependence	17
2.11. Constraints	17
2.12. Analytic Aspects	18
2.13. Marginal Returns	19
2.14. Linear Equations and Quadratic Criteria Exercises	20
2.15. Discussion Exercise	22

2.16. An Example of Constraints	23
2.17. Summary of Results	26
Exercises	26
2.18. Lagrange Parameter	27
2.19. Discussion	28
Exercises	29
2.20. Courant Parameter	29
Exercises	29
2.21. Infinite Processes	29
2.22. Approximation in Policy Space	30
2.23. Minimum of Maximum Deviation	31
Exercises	31
2.24. Discrete Stochastic Control Processes	31
Miscellaneous Exercises	33
Bibliography and Comments	37

Chapter 3 Computational Aspects of Dynamic Programming

3.1. Introduction	40
3.2. Discretization	41
3.3. Minimization	42
3.4. Storage Requirements: Discretion in the Use of Discretization	43
3.5. Time Requirements	45
3.6. Discussion	46
3.7. Simplified Policies	46
3.8. Stability	49
3.9. Computing as a Control Process	49
3.10. Stability Analysis	50
3.11. Functions and Solutions	51
Exercise	53
3.12. Interpolation	53
3.13. Computational Procedure	54
3.14. Evaluation of Polynomials	55
3.15. Orthogonal Polynomials and Quadrature	56
3.16. Polygonal Approximation	57
3.17. Adaptive Polygonal Approximation	58
3.18. Dynamic Programming Approach	59
Exercises	60
3.19. Linkage	61
Exercise	61
3.20. Search Processes	61
3.21. Discussion	63
3.22. Continuity	63
Exercise	64
3.23. Restriction to Grid Points	64
3.24. Multidimensional Case	65
Miscellaneous Exercises	65
Bibliography and Comments	66

Chapter 4 **Continuous Control Processes and the Calculus of Variations**

4.1. Introduction	70
4.2. The Quadratic Case	72
4.3. Discussion	73
4.4. Formal Derivation of the Euler Equation	74
4.5. Haar's Device	75
Exercises	76
4.6. Discussion	76
4.7. Nonexistence of a Minimum—I	77
Exercise	77
4.8. Nonexistence of a Minimum—II	77
Exercise	78
4.9. Nonexistence of a Minimum—III	78
4.10. Existence of Solution of Euler Equation	79
4.11. Successive Approximations	79
Exercises	81
4.12. Conditional Uniqueness	82
4.13. A Plethora of Geodesics	82
Exercises	83
4.14. A Priori Bounds	83
Exercises	84
4.15. Convexity	84
Exercises	84
4.16. Sufficient Condition for Absolute Minimum	84
4.17. Uniqueness of Solution of Euler Equation	86
4.18. Demonstration of Minimizing Property—Small T	86
4.19. Discussion	87
4.20. Solution as Function of Initial State	87
Exercises	89
4.21. Solution as Function of Duration of Process	89
4.22. The Return Function	90
4.23. A Nonlinear Partial Differential Equation	90
4.24. Discussion	91
Exercises	91
4.25. More General Control Processes	92
4.26. Discussion	94
Exercises	94
4.27. Multidimensional Control Processes—I	94
Exercise	95
4.28. Auxiliary Results	95
Exercises	96
4.29. Multidimensional Control Processes—II	96
Exercise	98
4.30. Functional Analysis	98
4.31. Existence and Uniqueness of Solution of Euler Equation	99
4.32. Global Constraints	100
Exercise	101

4.33. Necessity of Euler Equation	101
4.34. Minimization by Inequalities	102
Exercise	103
4.35. Inverse Problems	104
Miscellaneous Exercises	104
Bibliography and Comments	105

Chapter 5 Computational Aspects of the Calculus of Variations

5.1. Introduction	108
5.2. Successive Approximations and Storage	108
5.3. Circumvention of Storage	110
5.4. Discussion	112
5.5. Functions and Algorithms	113
5.6. Quasilinearization	114
5.7. Convergence	115
5.8. Discussion	117
5.9. Judicious Choice of Initial Approximation	117
5.10. Circumvention of Storage	118
5.11. Multidimensional Case	118
5.12. Two-point Boundary-value Problems	119
5.13. Analysis of Computational Procedure	120
5.14. Instability	120
5.15. Dimensionality	121
5.16. Matrix Inversion	122
5.17. Tychonov Regularization	122
5.18. Quadrature Techniques	124
5.19. "The Proof Is in the Program"	125
5.20. Numerical Solution of Nonlinear Euler Equations	126
5.21. Interpolation and Search	126
5.22. Extrapolation	127
5.23. Bubnov-Galerkin Method	128
5.24. Method of Moments	129
5.25. Gradient Methods	130
5.26. A Specific Example	131
5.27. Numerical Procedure: Semi-discretization	132
5.28. Nonlinear Extrapolation	133
5.29. Rayleigh-Ritz Methods	134
Miscellaneous Exercises	134
Bibliography and Comments	135

Chapter 6 Continuous Control Processes and Dynamic Programming

6.1. Introduction	138
6.2. Continuous Multistage Decision Processes	139
6.3. Duality	140

6.4.	Analytic Formalism	140
6.5.	Limiting Form	142
6.6.	Discussion	142
6.7.	Associated Nonlinear Partial Differential Equations	142
6.8.	Characteristics and the Euler Equation	144
6.9.	Rigorous Aspects	144
6.10.	Multidimensional Case	146
6.11.	Riccati Equation	146
6.12.	Computational Significance	148
6.13.	Finite Difference Techniques	148
6.14.	Unconventional Difference Approximations	150
6.15.	Unconventional Difference Approximations Continued	150
6.16.	Power Series Expansions	151
6.17.	Perturbation Series	152
6.18.	Existence and Uniqueness	153
6.19.	Positivity	153
6.20.	Uniqueness	154
6.21.	Approximation in Policy Space	154
6.22.	Quasilinearization	155
6.23.	Representation of Solution of Partial Differential Equation	156
6.24.	Constraints	158
6.25.	Inverse Problems	158
6.26.	Semi-groups and the Calculus of Variations	160
	Miscellaneous Exercises	160
	Bibliography and Comments	163

Chapter 7 Limiting Behavior of Discrete Processes

7.1.	Introduction	167
7.2.	Discrete Approximation to the Continuous and Conversely	168
7.3.	Suboptimization	168
7.4.	Lower Bound	169
7.5.	Further Reduction	170
7.6.	Linear Equations and Quadratic Criteria	170
7.7.	Sophisticated Quadrature	170
7.8.	Degree of Approximation	171
7.9.	Quadratic Case	172
7.10.	Convex Case	173
7.11.	Deferred Passage to the Limit	174
7.12.	Use of Analytic Structure	175
7.13.	Self-consistent Convergence	176
7.14.	Precise Formulation	176
7.15.	Lipschitz Conditions	177
7.16.	An Intermediate Process	178
7.17.	Comparison of g_n and h_{2n}	179
7.18.	$f_N(c, \Delta)$ as a Function of Δ	179
7.19.	Almost Monotone Convergence	179
7.20.	Discussion	180
	Bibliography and Comments	180

Chapter 8 Asymptotic Control Theory

8.1. Introduction	181
8.2. Asymptotic Control	182
8.3. Existence of Limit of $f(c, T)$	183
8.4. Poincare-Lyapunov Theory	184
8.5. Analogous Result for Two-point Boundary-value Problem	185
8.6. The Associated Green's Function	186
8.7. Conversion to Integral Equation	188
8.8. Conditional Uniqueness	189
8.9. Asymptotic Behavior of u —I	190
8.10. Asymptotic Behavior of u —II	190
8.11. Infinite Control Process	192
8.12. Multidimensional Case	192
8.13. Asymptotic Control	193
8.14. Boundedness of $\ x(t)\ $	193
8.15. Convergence of $f(c, T)$	194
8.16. Conclusion of Proof	195
8.17. Infinite Processes	195
8.18. Discrete Infinite Processes	196
8.19. Steady-state Average Behavior	197
8.20. Subadditive Functions	198
8.21. Proof of Theorem Bibliography and Comments	199
	201

Chapter 9 Duality and Upper and Lower Bounds

9.1. Introduction	202
9.2. Formalism	203
9.3. Quadratic Case	204
9.4. Multidimensional Quadratic Case	206
9.5. Numerical Utilization	207
9.6. The Legendre-Fenchel Transform	208
9.7. Convex g	209
9.8. Multidimensional Legendre-Fenchel Transform	210
9.9. Convex $g(x)$	211
9.10. Alternate Approach	212
9.11. Duality	213
9.12. Upper and Lower Bounds for Partial Differential Equations	214
9.13. Upper Bounds for $f(c, T)$	214
9.14. Lower Bounds	215
9.15. Perturbation Technique	216
9.16. Convexity of h	217
9.17. $2g(c) = c^2$, h Convex	217
9.18. The Maximum Transform	218
9.19. Application to Allocation Processes	219
9.20. Multistage Allocation	220
9.21. General Maximum Convolution Miscellaneous Exercises	221
	222
Bibliography and Comments	224

Chapter 10 Abstract Control Processes and Routing

10.1.	Introduction	226
10.2.	The Routing Problem	227
10.3.	Dynamic Programming Approach	228
10.4.	Upper and Lower Bounds	229
10.5.	Existence and Uniqueness	231
10.6.	Optimal Policy	233
10.7.	Approximation in Policy Space	233
10.8.	Computational Feasibility	234
10.9.	Storage of Algorithms	235
10.10.	Alternate Approaches	235
10.11.	“Traveling Salesman” Problem	236
10.12.	Stratification	237
10.13.	Routing and Control Processes	238
10.14.	Computational Procedure	239
10.15.	Feasibility	240
10.16.	Perturbation Technique	241
10.17.	Generalized Routing	242
10.18.	Pawn-King Endings in Chess	243
10.19.	Discussion	244
	Bibliography and Comments	245

Chapter 11 Reduction of Dimensionality

11.1.	Introduction	248
11.2.	A Terminal Control Process	249
11.3.	Preliminary Transformation	250
11.4.	New State Variables	250
11.5.	Partial Differential Equation for $\varphi(z, T)$	251
11.6.	Discussion	252
11.7.	Riccati Differential Equation	252
11.8.	General Terminal Criterion	253
11.9.	Constraints	253
11.10.	Successive Approximations	254
11.11.	Quadratic Case	255
11.12.	A General Nonlinear Case	256
	Bibliography and Comments	257

Chapter 12 Distributed Control Processes and the Calculus of Variations

12.1.	Introduction	259
12.2.	A Heat Control Process	260
12.3.	The Euler Formalism	260
12.4.	Rigorous Aspects	262
12.5.	Laplace Transform	263
12.6.	An Integral Equation	265

12.7. The Variational Equation	266
12.8. Computational Approaches	266
12.9. Modified Liouville-Neumann Solution	266
12.10. Use of Difference Approximations	267
12.11. Generalized Quadrature	268
12.12. Orthogonal Expansion and Truncation	270
12.13. Differential-Difference Equation	271
12.14. The Euler Equation	271
12.15. Discussion	272
12.16. Laplace Transform	572
12.17. Integral Equation	274
12.18. Variational Equation	274
12.19. Discretization	275
12.20. Discussion	275
Miscellaneous Exercises	275
Bibliography and Comments	276

Chapter 13 **Distributed Control Processes and Dynamic Programming**

13.1. Introduction	278
13.2. Formulation of a General Control Process	278
13.3. Multistage Decision Process	279
13.4. Functional Derivative	279
13.5. Formal Derivation of Functional Equation	280
13.6. Discussion	281
13.7. Quadratic Criteria	281
13.8. Integral Equations	281
13.9. Expression for $\min J(u)$	282
13.10. Functional Equation for $f(v, a)$	283
13.11. The Form of $L(k(a, t), a)$	283
13.12. Functional Equation for $q(t, s, a)$	284
13.13. The Potential Equation	285
13.14. Reduction of Dimensionality—I	285
13.15. Reduction in Dimensionality—II	286
Miscellaneous Exercises	286
Bibliography and Comments	287

Chapter 14 **Some Directions of Research**

14.1. Introduction	289
14.2. Constraints	290
14.3. Control Takes Time	291
14.4. Principle of Macroscopic Uncertainty	292
14.5. “On-Line” Control	292
14.6. Monotone Approximation	293
14.7. Identification and Control	293
14.8. Mathematical Model-Making	294

14.9. Computing as a Control Process	294
14.10. Physiological Control Processes	294
14.11. Environmental Control	295
14.12. Classical and Nonclassical Mechanics	295
14.13. Two-Person and N -Person Processes	295
14.14. Conclusion	295
Bibliography and Comment	296
<i>Author Index</i>	299
<i>Subject Index</i>	304