

Signal Processing and Frequency-Wavenumber Spectrum Analysis for a Large Aperture Seismic Array*

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Models of the Sources of the Earth's Magnetic Field

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Computations with Spherical Harmonics and Fourier Series in Geomagnetism

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Inverse Methods in the Interpretation of Magnetic and Gravity Anomalies

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Analysis of Geoelectromagnetic Data

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Nonlinear Spherical Harmonic Analysis of Paleomagnetic Data

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Harmonic Analysis of Earth Tides

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Computer Usage in the Computation of Gravity Anomalies¹

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Analysis of Irregularities in the Earth's Rotation

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Convection in the Earth's Mantle

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