# Numerical Methods of Ray Generation in Multilayered Media

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# Computer Generated Seismograms

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# Inversion and Inference for Teleseismic Ray Data

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# Multipolar Analysis of the Mechanisms of Deep-Focus Earthquakes

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# Computation of Models of Elastic Dislocations in the Earth

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