

The Electrostatic Sheet Model for a Plasma and its Modification to Finite-Size Particles

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Solution of Vlasov's Equation by Transform Methods

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The Water-Bag Model*

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* This work was supported jointly by the U.S. Atomic Energy Commission and the U.K. Atomic Energy Authority.

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Multidimensional Plasma Simulation by the Particle-in-Cell Method*

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Finite-Size Particle Physics Applied to Plasma Simulation

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Finite-Difference Methods for Collisionless Plasma Models

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Application of Hamilton's Principle to the Numerical Analysis of Vlasov Plasmas[†]

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Magnetohydrodynamic Calculations

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The Solution of the Fokker-Planck Equation for a Mirror-Confined Plasma

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