

Two-Dimensional Lagrangian Hydrodynamic Difference Equations¹

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Mixed Eulerian-Lagrangian Method¹

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The Strip Code and the Jetting of Gas between Plates

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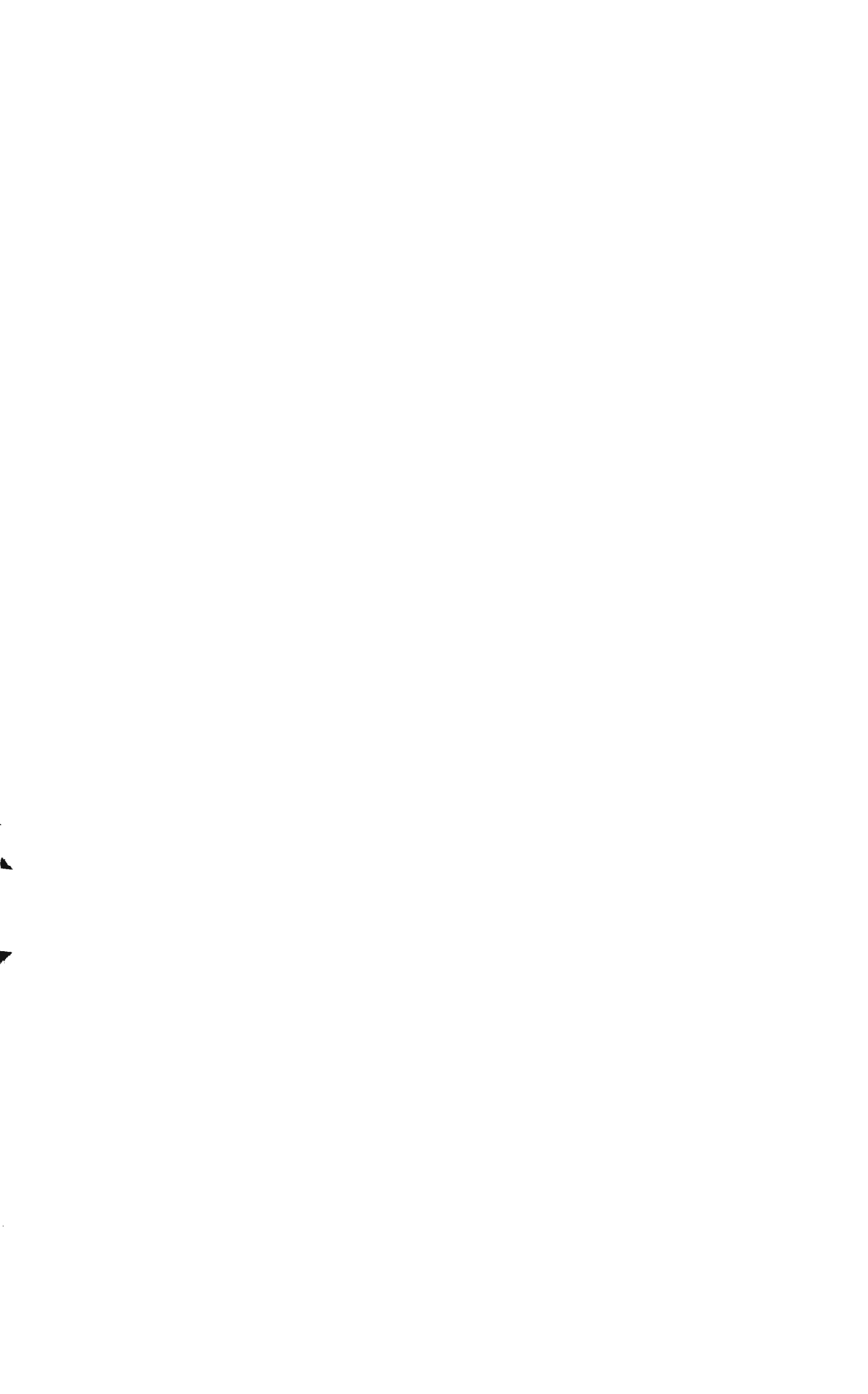
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CEL: A Time-Dependent, Two-Space-Dimensional, Coupled Eulerian-Lagrange Code*

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The TENSOR Code*

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Calculation of Elastic-Plastic Flow

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Solution by Characteristics of the Equations of One-Dimensional Unsteady Flow

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The Solution of Two-Dimensional Hydrodynamic Equations by the Method of Characteristics

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The Particle-in-Cell Computing Method for Fluid Dynamics

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The Time Dependent Flow of an Incompressible Viscous Fluid

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