# The Gaussian Function in Calculations of Statistical Mechanics and Quantum Mechanics

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# Atomic Self-Consistent Field Calculations by the Expansion Method

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# Nonseparable Theory of Electron-Hydrogen Scattering

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