

TABLE OF CONTENTS

GENERAL INTRODUCTION	1
<u>I. PRELIMINARIES</u>	8
1.0 INTRODUCTION.	8
1.1 BASIC MATHEMATICAL CONCEPTS	8
1.2 THE STOCHASTIC INTEGRAL EQUATIONS	15
1.3 STOCHASTIC DIFFERENTIAL SYSTEMS	25
<u>II. A RANDOM INTEGRAL EQUATION OF THE VOLTERRA TYPE</u>	27
2.0 INTRODUCTION.	27
2.1 SOME PRELIMINARY LEMMAS	28
2.2 EXISTENCE AND UNIQUENESS OF A RANDOM SOLUTION OF (2.0.1)	30
2.3 SOME SPECIAL CASES.	34
2.4 ASYMPTOTIC BEHAVIOR OF THE RANDOM SOLUTION.	41
2.5 APPLICATION TO THE POINCARÉ-LYAPUNOV STABILITY THEOREM	43
<u>III. APPROXIMATE SOLUTIONS OF THE RANDOM VOLTERRA INTEGRAL EQUATION</u>	48
3.0 INTRODUCTION.	48
3.1 THE METHOD OF SUCCESSIVE APPROXIMATIONS	49
3.1.1 Almost Sure Convergence of Successive Approximations	51
3.1.2 Rate of Convergence and Mean Square Error of Approximation.	56

3.1.3 Combined Error of Approximation and Numerical Integration.	59
3.2 THE METHOD OF STOCHASTIC APPROXIMATION.	66
3.2.1 A Stochastic Approximation Procedure	66
3.2.2 Solution of the Random Volterra Equation by Stochastic Approximation.	68
 <u>IV. A STOCHASTIC INTEGRAL EQUATION OF THE FREDHOLM TYPE WITH</u>	
<u>APPLICATION TO SYSTEMS THEORY.</u>	76
4.0 INTRODUCTION.	76
4.1 EXISTENCE AND UNIQUENESS OF A RANDOM SOLUTION	77
4.2 SOME SPECIAL CASES.	92
4.3 STOCHASTIC ASYMPTOTIC STABILITY OF THE RANDOM SOLUTION.	96
4.4 AN APPLICATION IN STOCHASTIC CONTROL SYSTEMS.	99
 <u>V. RANDOM DISCRETE FREDHOLM AND VOLTERRA EQUATIONS.</u> 106	
5.0 INTRODUCTION.	106
5.1 EXISTENCE AND UNIQUENESS OF A RANDOM SOLUTION	107
5.2 SPECIAL CASES OF THEOREM 5.1.3.	112
5.3 STOCHASTIC STABILITY OF THE RANDOM SOLUTION	116
5.4 APPLICATION TO STOCHASTIC SYSTEMS	120
 <u>VI. THE STOCHASTIC DIFFERENTIAL SYSTEMS</u>	
$\dot{x}(t; \omega) = A(\omega)x(t; \omega) + b(\omega)\phi(\sigma(t; \omega))$ WITH	
$\sigma(t; \omega) = \langle c(t; \omega), x(t; \omega) \rangle$ AND	
$\dot{x}(t; \omega) = A(\omega)x(t; \omega) + b(\omega)\phi(\sigma(t; \omega))$ WITH	
$\sigma(t; \omega) = f(t; \omega) + \int_0^t \langle c(t-\tau; \omega), x(\tau; \omega) \rangle d\tau$	130

VII

6.0	INTRODUCTION.	130
6.1	REDUCTION OF THE STOCHASTIC DIFFERENTIAL SYSTEMS.	132
6.2	STOCHASTIC ABSOLUTE STABILITY OF THE SYSTEMS.	134

VII. THE STOCHASTIC DIFFERENTIAL SYSTEMS

$$\dot{\mathbf{x}}(t; \omega) = \mathbf{A}(\omega)\mathbf{x}(t; \omega) + \int_0^t \mathbf{b}(t-\eta; \omega)\phi(\sigma(\eta; \omega))d\eta$$

WITH

$$\sigma(t; \omega) = f(t; \omega) + \int_0^t c(t-\eta; \omega), x(\eta; \omega) d\eta$$

AND

$$\begin{aligned}\dot{\mathbf{x}}(t; \omega) &= A(\omega) \mathbf{x}(t; \omega) + \int_0^t b(t-\tau; \omega) \phi(\sigma(\tau; \omega)) d\tau \\ &\quad + \int_0^t c(t-\tau; \omega) \sigma(\tau; \omega) d\tau\end{aligned}$$

WITH

$$\sigma(t; \omega) = f(t; \omega) + \int_0^t \langle d(t-\tau; \omega), x(\tau; \omega) \rangle d\tau \quad \quad 144$$

7.0	INTRODUCTION.	144
7.1	REDUCTION OF THE STOCHASTIC DIFFERENTIAL SYSTEMS.	145
7.2	STOCHASTIC ABSOLUTE STABILITY OF THE SYSTEMS.	149

VIII. THE STOCHASTIC DIFFERENTIAL SYSTEMS WITH LAG TIME

$$\dot{\mathbf{x}}(t; \omega) = A(\omega)\mathbf{x}(t; \omega) + B(\omega)\mathbf{x}(t-\tau; \omega) + b(\omega)\phi(\sigma(t; \omega))$$

WITH

$$\sigma(t; \omega) = f(t; \omega) + \int_0^t c(t-s; \omega), x(s; \omega) ds$$

AND

$$\begin{aligned}\dot{x}(t; \omega) &= A(\omega)x(t; \omega) + B(\omega)x(t-\tau; \omega) \\ &\quad + \int_0^t \eta(t-u; \omega)\phi(\sigma(u; \omega))du + b(\omega)\phi(\sigma(t; \omega))\end{aligned}$$

WITH

8.0	INTRODUCTION.	156
8.1	REDUCTION OF THE STOCHASTIC DIFFERENTIAL SYSTEMS.	158
8.2	STOCHASTIC ABSOLUTE STABILITY OF THE SYSTEMS.	161