

Contents

CHAPTER 1

Algebra of Vectors and Matrices	1
1a. Vector Spaces	2
<i>1a.1 Definition of Vector Spaces and Subspaces, 1a.2 Basis of a Vector Space, 1a.3 Linear Equations, 1a.4 Vector Spaces with an Inner Product</i>	
Complements and Problems	11
1b. Theory of Matrices and Determinants	14
<i>1b.1 Matrix Operations, 1b.2 Elementary Matrices and Diagonal Reduction of a Matrix, 1b.3 Determinants, 1b.4 Transformations 1b.5 Generalized Inverse of a Matrix, 1b.6 Matrix Representation, of Vector Spaces, Bases, etc., 1b.7 Idempotent Matrices, 1b.8 Special Products of Matrices</i>	
Complements and Problems	30
1c. Eigenvalues and Reduction of Matrices	34
<i>1c.1 Classification and Transformation of Quadratic Forms, 1c.2 Roots of Determinantal Equations, 1c.3 Canonical Reduction of Matrices, 1c.4 Projection Operator, 1c.5 Further Results on g-Inverse, 1c.6 Restricted Eigenvalue Problem</i>	
1d. Convex Sets in Vector Spaces	51
<i>1d.1 Definitions, 1d.2 Separation Theorems for Convex Sets</i>	

1e.	Inequalities	53
	<i>1e.1 Cauchy-Schwarz (C-S) Inequality, 1e.2 Hölder's Inequality, 1e.3 Hadamard's Inequality, 1e.4 Inequalities Involving Moments, 1e.5 Convex Functions and Jensen's Inequality, 1e.6 Inequalities in Information Theory, 1e.7 Stirling's Approximation</i>	
1f.	Extrema of Quadratic Forms	60
	<i>1f.1 General Results, 1f.2 Results Involving Eigenvalues and Vectors 1f.3 Minimum Trace Problems</i>	
	Complements and Problems	67
CHAPTER 2		
	Probability Theory, Tools and Techniques	79
2a.	Calculus of Probability	80
	<i>2a.1 The Space of Elementary Events, 2a.2 The Class of Subsets (Events), 2a.3 Probability as a Set Function, 2a.4 Borel Field (σ-field) and Extension of Probability Measure, 2a.5 Notion of a Random Variable and Distribution Function, 2a.6 Multidimensional Random Variable, 2a.7 Conditional Probability and Statistical Independence, 2a.8 Conditional Distribution of a Random Variable</i>	
2b.	Mathematical Expectation and Moments of Random Variables	92
	<i>2b.1 Properties of Mathematical Expectation, 2b.2 Moments, 2b.3 Conditional Expectation, 2b.4 Characteristic Function (c.f.), 2b.5 Inversion Theorems, 2b.6 Multivariate Moments</i>	
2c.	Limit Theorems	108
	<i>2c.1 Kolmogorov Consistency Theorem, 2c.2 Convergence of a Sequence of Random Variables, 2c.3 Law of Large Numbers, 2c.4 Convergence of a Sequence of Distribution Functions, 2c.5 Central Limit Theorems, 2c.6 Sums of Independent Random Variables</i>	
2d.	Family of Probability Measures and Problems of Statistics	130
	<i>2d.1 Family of Probability Measures, 2d.2 The Concept of a Sufficient Statistic, 2d.3 Characterization of Sufficiency</i>	
	Appendix 2A. Stieltjes and Lebesgue Integrals	132
	Appendix 2B. Some Important Theorems in Measure Theory and Integration	134

Appendix 2C. Invariance	138
Appendix 2D. Statistics, Subfields, and Sufficiency	139
Appendix 2E. Non-Negative Definiteness of a Characteristic Function	141
Complements and Problems	142

CHAPTER 3

Continuous Probability Models	155
3a. Univariate Models	158
<i>3a.1 Normal Distribution, 3a.2 Gamma Distribution, 3a.3 Beta Distribution, 3a.4 Cauchy Distribution, 3a.5 Student's t Distribution, 3a.6 Distributions Describing Equilibrium States in Statistical Mechanics, 3a.7 Distribution on a Circle</i>	
3b. Sampling Distributions	179
<i>3b.1 Definitions and Results, 3b.2 Sum of Squares of Normal Variables, 3b.3 Joint Distribution of the Sample Mean and Variance, 3b.4 Distribution of Quadratic Forms, 3b.5 Three Fundamental Theorems of the Least Squares Theory, 3b.6 The p-Variate Normal Distribution, 3b.7 The Exponential Family of Distributions</i>	
3c. Symmetric Normal Distribution	197
<i>3c.1 Definition, 3c.2 Sampling Distributions</i>	
3d. Bivariate Normal Distribution	201
<i>3d.1 General Properties, 3d.2 Sampling Distributions</i>	
Complements and Problems	209

CHAPTER 4

The Theory of Least Squares and Analysis of Variance	220
4a. Theory of Least Squares (Linear Estimation)	221
<i>4a.1 Gauss-Markoff Setup ($Y, X\beta, \sigma^2 I$), 4a.2 Normal Equations and Least Squares (l.s.) Estimators, 4a.3 g-Inverse and a Solution of the Normal Equation, 4a.4 Variances and Covariances of l.s. Estimators, 4a.5 Estimation of σ^2, 4a.6 Other Approaches to the l.s. Theory (Geometric Solution), 4a.7 Explicit Expressions for Correlated Observations, 4a.8 Some Computational Aspects of the l.s. Theory, 4a.9 Least Squares Estimation with Restrictions on Parameters,</i>	

<i>4a.10 Simultaneous Estimation of Parametric Functions,</i>	<i>4a.11 Least Squares Theory when the Parameters Are Random Variables,</i>	<i>4a.12 Choice of the Design Matrix</i>	
4b. Tests of Hypotheses and Interval Estimation			236
<i>4b.1 Single Parametric Function (Inference),</i>	<i>4b.2 More than One Parametric Function (Inference),</i>	<i>4b.3 Setup with Restrictions</i>	
4c. Problems of a Single Sample			243
<i>4c.1 The Test Criterion,</i>	<i>4c.2 Asymmetry of Right and Left Femora (Paired Comparison)</i>		
4d. One-Way Classified Data			244
<i>4d.1 The Test Criterion,</i>	<i>4d.2 An Example</i>		
4e. Two-Way Classified Data			247
<i>4e.1 Single Observation in Each Cell,</i>	<i>4e.2 Multiple but Equal Numbers in Each Cell,</i>	<i>4e.3 Unequal Numbers in Cells</i>	
4f. A General Model for Two-Way Data and Variance Components			258
<i>4f.1 A General Model,</i>	<i>4f.2 Variance Components Model,</i>	<i>4f.3 Treatment of the General Model</i>	
4g. The Theory and Application of Statistical Regression			263
<i>4g.1 Concept of Regression (General Theory),</i>	<i>4g.2 Measurement of Additional Association,</i>	<i>4g.3 Prediction of Cranial Capacity (a Practical Example),</i>	<i>4g.4 Test for Equality of the Regression Equations,</i>
<i>4g.5 The Test for an Assigned Regression Function,</i>	<i>4g.6 Restricted Regression</i>		
4h. The General Problem of Least Squares with Two Sets of Parameters			288
<i>4h.1 Concomitant Variables,</i>	<i>4h.2 Analysis of Covariance,</i>	<i>4h.3 An Illustrative Example</i>	
4i. Unified Theory of Linear Estimation			294
<i>4i.1 A Basic Lemma on Generalized Inverse,</i>	<i>4i.2 The General Gauss-Markoff Model (GGM),</i>	<i>4i.3 The Inverse Partitioned Matrix (IPM) Method,</i>	<i>4i.4 Unified Theory of Least Squares</i>
4j. Estimation of Variance Components			302
<i>4j.1 Variance Components Model,</i>	<i>4j.2 MINQUE Theory,</i>	<i>4j.3 Computation under the Euclidian Norm</i>	

4k. Biased Estimation in Linear Models	305
4k.1 <i>Best Linear Estimator (BLE)</i> , 4k.2 <i>Best Linear Minimum Bias Estimation (BLIMBE)</i>	
Complements and Problems	308

CHAPTER 5

Criteria and Methods of Estimation	314
5a. Minimum Variance Unbiased Estimation	315
5a.1 <i>Minimum Variance Criterion</i> , 5a.2 <i>Some Fundamental Results on Minimum Variance Estimation</i> , 5a.3 <i>The Case of Several Parameters</i> , 5a.4 <i>Fisher's Information Measure</i> , 5a.5 <i>An Improvement of Unbiased Estimators</i>	
5b. General Procedures	334
5b.1 <i>Statement of the General Problem (Bayes Theorem)</i> , 5b.2 <i>Joint d.f. of (θ, x) Completely Known</i> , 5b.3 <i>The Law of Equal Ignorance</i> , 5b.4 <i>Empirical Bayes Estimation Procedures</i> , 5b.5 <i>Fiducial Probability</i> , 5b.6 <i>Minimax Principle</i> , 5b.7 <i>Principle of Invariance</i>	
5c. Criteria of Estimation in Large Samples	344
5c.1 <i>Consistency</i> , 5c.2 <i>Efficiency</i>	
5d. Some Methods of Estimation in Large Samples	351
5d.1 <i>Method of Moments</i> , 5d.2 <i>Minimum Chi-Square and Associated Methods</i> , 5d.3 <i>Maximum Likelihood</i>	
5e. Estimation of the Multinomial Distribution	355
5e.1 <i>Nonparametric Case</i> , 5e.2 <i>Parametric Case</i>	
5f. Estimation of Parameters in the General Case	363
5f.1 <i>Assumptions and Notations</i> , 5f.2 <i>Properties of m.l. Equation Estimators</i>	
5g. The Method of Scoring for the Estimation of Parameters	366
Complements and Problems	374

CHAPTER 6

Large Sample Theory and Methods	382
6a. Some Basic Results	382
6a.1 <i>Asymptotic Distribution of Quadratic Functions of Frequencies</i> , 6a.2 <i>Some Convergence Theorems</i>	

6b.	Chi-Square Tests for the Multinomial Distribution	390
	<i>6b.1 Test of Departure from a Simple Hypothesis, 6b.2 Chi-Square Test for Goodness of Fit, 6b.3 Test for Deviation in a Single Cell, 6b.4 Test Whether the Parameters Lie in a Subset, 6b.5 Some Examples, 6b.6 Test for Deviations in a Number of Cells</i>	
6c.	Tests Relating to Independent Samples from Multinomial Distributions	398
	<i>6c.1 General Results, 6c.2 Test of Homogeneity of Parallel Samples, 6c.3 An Example</i>	
6d.	Contingency Tables	403
	<i>6d.1 The Probability of an Observed Configuration and Tests in Large Samples, 6d.2 Tests of Independence in a Contingency Table, 6d.3 Tests of Independence in Small Samples</i>	
6e.	Some General Classes of Large Sample Tests	415
	<i>6e.1 Notations and Basic Results, 6e.2 Test of a Simple Hypothesis, 6e.3 Test of a Composite Hypothesis</i>	
6f.	Order Statistics	420
	<i>6f.1 The Empirical Distribution Function, 6f.2 Asymptotic Distribution of Sample Fractiles</i>	
6g.	Transformation of Statistics	426
	<i>6g.1 A General Formula, 6g.2 Square Root Transformation of the Poisson Variate, 6g.3 \sin^{-1} Transformation of the Square Root of the Binomial Proportion, 6g.4 \tanh^{-1} Transformation of the Correlation Coefficient</i>	
6h.	Standard Errors of Moments and Related Statistics	436
	<i>6h.1 Variances and Covariances of Raw Moments, 6h.2 Asymptotic Variances and Covariances of Central Moments, 6h.3 Exact Expressions for Variances and Covariances of Central Moments</i>	
	Complements and Problems	439
CHAPTER 7		
	Theory of Statistical Inference	444
7a.	Testing of Statistical Hypotheses	445
	<i>7a.1 Statement of the Problem, 7a.2 Neyman-Pearson Fundamental Lemma and Generalizations, 7a.3 Simple H_0 against Simple H_1</i>	

7a.4 Locally Most Powerful Tests,	7a.5 Testing a Composite Hypothesis,	7a.6 Fisher-Behrens Problem,	7a.7 Asymptotic Efficiency of Tests	
7b. Confidence Intervals				470
7b.1 The General Problem,	7b.2 A General Method of Constructing a Confidence Set,	7b.3 Set Estimators for Functions of θ		
7c. Sequential Analysis				474
7c.1 Wald's Sequential Probability Ratio Test,	7c.2 Some Properties of the S.P.R.T.,	7c.3 Efficiency of the S.P.R.T.,	7c.4 An Example of Economy of Sequential Testing,	7c.5 The Fundamental Identity of Sequential Analysis,
7c.6 Sequential Estimation,	7c.7 Sequential Tests with Power One			
7d. Problem of Identification—Decision Theory				491
7d.1 Statement of the Problem,	7d.2 Randomized and Nonrandomized Decision Rules,	7d.3 Bayes Solution,	7d.4 Complete Class of Decision Rules,	7d.5 Minimax Rule
7e. Nonparametric Inference				497
7e.1 Concept of Robustness,	7e.2 Distribution-Free Methods,	7e.3 Some Nonparametric Tests,	7e.4 Principle of Randomization	
7f. Ancillary Information				505
Complements and Problems				506
CHAPTER 8				
Multivariate Analysis				516
8a. Multivariate Normal Distribution				517
8a.1 Definition,	8a.2 Properties of the Distribution,	8a.3 Some Characterizations of N_p ,	8a.4 Density Function of the Multivariate Normal Distribution,	8a.5 Estimation of Parameters,
8a.6 N_p as a Distribution with Maximum Entropy				
8b. Wishart Distribution				533
8b.1 Definition and Notation,	8b.2 Some Results on Wishart Distribution			
8c. Analysis of Dispersion				543
8c.1 The Gauss-Markoff Setup for Multiple Measurements,	8c.2 Estimation of Parameters,	8c.3 Tests of Linear Hypotheses, Analysis of		

xx CONTENTS

Dispersion (A.D.), 8c.4 Test for Additional Information, 8c.5 The Distribution of Λ , 8c.6 Test for Dimensionality (Structural Relationship), 8c.7 Analysis of Dispersion with Structural Parameters (Growth Model)

8d.	Some Applications of Multivariate Tests	562
	<i>8d.1 Test for Assigned Mean Values, 8d.2 Test for a Given Structure of Mean Values, 8d.3 Test for Differences between Mean Values of Two Populations, 8d.4 Test for Differences in Mean Values between Several Populations, 8d.5 Barnard's Problem of Secular Variations in Skull Characters</i>	
8e.	Discriminatory Analysis (Identification)	574
	<i>8e.1 Discriminant Scores for Decision, 8e.2 Discriminant Analysis in Research Work, 8e.3 Discrimination between Composite Hypotheses</i>	
8f.	Relation between Sets of Variates	582
	<i>8f.1 Canonical Correlations, 8f.2 Properties of Canonical Variables, 8f.3 Effective Number of Common Factors, 8f.4 Factor Analysis</i>	
8g.	Orthonormal Basis of a Random Variable	587
	<i>8g.1 The Gram-Schmidt Basis, 8g.2 Principal Component Analysis</i>	
	Complements and Problems	593
	Publications of the Author	605
	Author Index	615
	Subject Index	618

