



# Table of Contents

## 1. Some Basic Identities and Inequalities

1.0	Objectives; Notation	1
1.1	Elementary Matrices	3
1.2	Some Factorizations	4
1.3	Projections, and the General Reciprocal	8
1.4	Some Determinantal Identities	10
1.5	The Lanczos Algorithm for Tridiagonalization	17
1.6	Orthogonal Polynomials	24
	<i>References</i>	27
	<i>Problems and Exercises</i>	29

## 2. Norms, Bounds, and Convergence

2.0	The Notion of a Norm	37
2.1	Convex Sets and Convex Bodies	38
2.2	Norms and Bounds	39
2.3	Norms, Bounds, and Spectral Radii	45
2.4	Nonnegative Matrices	48
2.5	Convergence; Functions of Matrices	53
	<i>References</i>	55
	<i>Problems and Exercises</i>	57

## 3. Localization Theorems and Other Inequalities

3.0	Basic Definitions	64
3.1	Exclusion Theorems	65
3.2	Inclusion and Separation Theorems	70
3.3	Minimax Theorems and the Field of Values	74
3.4	Inequalities of Wielandt and Kantovich	81
	<i>References</i>	84
	<i>Problems and Exercises</i>	85
		ix

**4. The Solution of Linear Systems: Methods of Successive Approximation**

4.0	Direct Methods and Others	91
4.1	The Inversion of Matrices	94
4.2	Methods of Projection	98
4.3	Norm-Reducing Methods	103
	<i>References</i>	115
	<i>Problems and Exercises</i>	116

**5. Direct Methods of Inversion**

5.0	Uses of the Inverse	122
5.1	The Method of Modification	123
5.2	Triangularization	125
5.3	A More General Formulation	131
5.4	Orthogonal Triangularization	133
5.5	Orthogonalization	134
5.6	Orthogonalization and Projection	137
5.7	The Method of Conjugate Gradients	139
	<i>References</i>	141
	<i>Problems and Exercises</i>	142

**6. Proper Values and Vectors:  
Normalization and Reduction of the Matrix**

6.0	Purpose of Normalization	147
6.1	The Method of Krylov	149
6.2	The Weber-Voetter Method	151
6.3	The Method of Danilevskii	156
6.4	The Hessenberg and the Lanczos Reductions	158
6.5	Proper Values and Vectors	163
6.6	The Method of Samuelson and Bryan	165
6.7	The Method of Leverrier	166
6.8	Deflation	168
	<i>References</i>	172
	<i>Problems and Exercises</i>	172

**7. Proper Values and Vectors: Successive Approximation**

7.0	Methods of Successive Approximation	178
7.1	The Method of Jacobi	179

7.2	The Method of Collar and Jahn	181
7.3	Powers of a Matrix	182
7.4	Simple Iteration (the Power Method)	187
7.5	Multiple Roots and Principal Vectors	190
7.6	Staircase Iteration (Treppeniteration)	191
7.7	The LR-Transformation	194
7.8	Bi-iteration	196
7.9	The QR-Transformation	197
	<i>References</i>	198
	<i>Problems and Exercises</i>	199

<i>BIBLIOGRAPHY</i>	203
---------------------	-----

<i>INDEX</i>	249
--------------	-----