



# *Contents*

PREFACE	xi
NOTATION	xiii

## **Chapter 1 Introduction and Certain Double Sampling Procedures**

1.1 Introduction to Sequential Testing	1
1.2 Sampling Inspection	3
1.3 Loss and Risk Functions	6
1.4 Certain Double Sampling Procedures	7

## **Chapter 2 The Sequential Probability Ratio Test**

2.1 The Sequential Probability Ratio Test (SPRT)	14
2.2 Finite Termination of the SPRT	23
2.3 The Operating Characteristic Function (OC Function)	25
2.4 Average Sample Number	28
2.5 Wald's Fundamental Identity	40
2.6 Bounds for the Expected Sample Size in a SPRT	54
2.7 Formulas for Calculating the Operating Characteristic and the Average Sample Number of Some Sequential Tests	67
2.8 Truncated SPRT	76
2.9 Optimal Properties of the SPRT	78
2.10 Generalized SPRT (GSPRT)	87

2.11 Restricted SPRT	89
2.12 Extended SPRTs	93
2.13 Asymptotic Properties of SPRTs	97

### Chapter 3 Sequential Tests for Composite Hypotheses

3.1 Wald's (1947) Method of Weight Functions	101
3.2 Sequential $t$ - and $t^2$ -Tests and Sequential Analogues of Stein's Two-Stage Test	103
3.3 Sequential $F$ -Test	119
3.4 Certain Two-Sample Sequential Procedures	121
3.5 Fraser Sufficiency and Conditional Test	123
3.6 Invariant Sequential Procedures	126
3.7 Sequential Likelihood Ratio Test Procedures	147
3.8 Sequential Tests between Three Hypotheses	163
3.9 Sequential $k$ -Decision Problems	175
3.10 Sequential Tests for Two-Sided Alternatives	195
3.11 Efficiency of the SPRT	209
3.12 Bayes Sequential Procedures	227
3.13 Iterated Logarithm Inequalities and Their Application to Sequential Testing Hypotheses	244
3.14 Certain Nonparametric Sequential Test Procedures	253
3.15 Locally Most Powerful (LMP) Sequential Tests	284

### Chapter 4 Sequential Estimation

4.1 Introductory Remarks	293
4.2 Certain Two-Stage Procedures	293
4.3 Formulation of the Problem of Sequential Estimation by Intervals or Sets	315
4.4 Wald's Sequential Estimation Procedures	316
4.5 Certain General Concepts of Sequential Estimation	317
4.6 Cramér–Rao Lower Bound for the Sequential Case	317
4.7 Sufficiency and Completeness in the Sequential Case	322
4.8 Minimax Estimation	332
4.9 A Class of Situations Where Bayes Sequential Estimation Procedure Is Nonsequential	343
4.10 Sequential Estimation and Certain Closed Sequential Decision Procedures	358
4.11 Estimation by Double Sampling	367
4.12 Large Sample Theory of Sequential Estimation	377
4.13 Sequential Confidence Intervals for the Mean of a Normal Population with Unknown Variance	385
4.14 The Asymptotic Theory of Fixed-Width Sequential Confidence Intervals for the Mean	412
4.15 Asymptotic Theory of Fixed-Size Sequential Confidence Bounds for Linear Regression Parameters	428
4.16 Confidence Sequences for Other Parameters	435
4.17 Sequential Estimation of the Mean Vector of a Multivariate Normal Distribution	452

4.18 A General Method of Determining Fixed-Width Confidence Intervals	464
4.19 Sequential Estimation of the Size of a Finite Population	470
4.20 Consistency of Certain Sequential Estimators	475
4.21 Asymptotically Optimal Bayes Sequential Estimation	482
4.22 Sequential Nonparametric Confidence Intervals for Certain Parameters of the Population	487
4.23 Certain Problems of Optimal Stopping	504
Appendix 1 <b>Solution to Wald's Equation</b>	513
Appendix 2 <b>Differentiation of an Expectation</b>	515
Appendix 3 <b>On the Moments of a Random Variable</b>	518
Appendix 4 <b>The Normal Diffusion Process</b>	520
Appendix 5 <b>Large Sample Properties of Maximum Likelihood Estimates</b>	527
Appendix 6 <b>A Table of Standard Distributions</b>	530
Appendix 7 <b>A Useful Convergence Theorem of Cramér</b>	532
Appendix 8 <b>Backward Induction</b>	534
REFERENCES AND AUTHOR INDEX	537
SUBJECT INDEX	555