

Contents

CHAPTER	PAGE
INTRODUCTION: THE NATURE OF PROBABILITY THEORY	1
1. The Background	1
2. Procedure	3
3. "Statistical" Probability	4
4. Summary	5
5. Historical Note	6
 I THE SAMPLE SPACE	 7
1. The Empirical Background	7
2. Examples	9
3. The Sample Space. Events	13
4. Relations among Events	15
5. Discrete Sample Spaces	17
6. Probabilities in Discrete Sample Spaces: Preparations	19
7. The Basic Definitions and Rules	22
8. Problems for Solution	24
 II ELEMENTS OF COMBINATORIAL ANALYSIS	 26
1. Preliminaries	26
2. Ordered Samples	28
3. Examples	30
4. Subpopulations and Partitions	32
*5. Application to Occupancy Problems	36
*5a. Application to Runs	40
6. The Hypergeometric Distribution	41
7. Examples for Waiting Times	45
8. Binomial Coefficients	48
9. Stirling's Formula	50
Problems for Solution:	
10. Exercises and Examples	53

* Starred sections are not required for the understanding of the sequel and should be omitted at first reading.

CHAPTER	PAGE
11. Problems and Complements of a Theoretical Character	57
12. Problems and Identities Involving Binomial Coefficients	61
*III FLUCTUATIONS IN COIN TOSSING AND RANDOM WALKS	65
1. General Orientation	66
2. Problems of Arrangements	69
3. Random Walks and Coin Tossing	73
4. Reformulation of the Combinatorial Theorems	74
5. Probability of Long Leads: The First Arc Sine Law	77
6. The Number of Returns to the Origin	81
7. An Experimental Illustration	83
8. Miscellaneous Complements	85
*IV COMBINATION OF EVENTS	88
1. Union of Events	88
2. Application to the Classical Occupancy Problem	91
3. The Realization of m among N Events	96
4. Application to Matching and Guessing	97
5. Miscellany	99
6. Problems for Solution	101
V CONDITIONAL PROBABILITY. STOCHASTIC INDEPENDENCE	104
1. Conditional Probability	104
2. Probabilities Defined by Conditional Probabilities. Urn Models	108
3. Stochastic Independence	114
4. Repeated Trials	118
*5. Applications to Genetics	121
*6. Sex-Linked Characters	125
*7. Selection	128
8. Problems for Solution	129
VI THE BINOMIAL AND THE POISSON DISTRIBUTIONS	135
1. Bernoulli Trials	135
2. The Binomial Distribution	136
3. The Central Term and the Tails	139
4. The Law of Large Numbers	141
5. The Poisson Approximation	142
6. The Poisson Distribution	146
7. Observations Fitting the Poisson Distribution	149

CHAPTER	PAGE
8. Waiting Times. The Negative Binomial Distribution	155
9. The Multinomial Distribution	157
10. Problems for Solution	158
VII THE NORMAL APPROXIMATION TO THE BINOMIAL	
DISTRIBUTION	164
1. The Normal Distribution	164
2. The DeMoivre-Laplace Limit Theorem	168
3. Examples	174
4. Relation to the Poisson Approximation	176
5. Large Deviations	178
6. Problems for Solution	179
*VIII UNLIMITED SEQUENCES OF BERNOULLI TRIALS	183
1. Infinite Sequences of Trials	183
2. Systems of Gambling	185
3. The Borel-Cantelli Lemmas	188
4. The Strong Law of Large Numbers	189
5. The Law of the Iterated Logarithm	191
6. Interpretation in Number Theory Language	195
7. Problems for Solution	197
IX RANDOM VARIABLES; EXPECTATION	199
1. Random Variables	199
2. Expectations	207
3. Examples and Applications	209
4. The Variance	213
5. Covariance; Variance of a Sum	215
6. Chebyshev's Inequality	219
*7. Kolmogorov's Inequality	220
*8. The Correlation Coefficient	221
9. Problems for Solution	223
X LAWS OF LARGE NUMBERS	228
1. Identically Distributed Variables	228
*2. Proof of the Law of Large Numbers	231
3. The Theory of "Fair" Games	233
*4. The Petersburg Game	235
5. Variable Distributions	238
*6. Applications to Combinatorial Analysis	241
*7. The Strong Law of Large Numbers	243
8. Problems for Solution	245

CHAPTER	PAGE
XI INTEGRAL VALUED VARIABLES. GENERATING FUNCTIONS	248
1. Generalities	248
2. Convolutions	250
3. Application to First Passage and Recurrence Times in Bernoulli Trials	254
4. Partial Fraction Expansions	257
5. Bivariate Generating Functions	261
*6. The Continuity Theorem	262
7. Problems for Solution	264
*XII COMPOUND DISTRIBUTIONS. BRANCHING PROCESSES	268
1. Sums of a Random Number of Variables	268
2. The Compound Poisson Distribution	270
3. Infinitely Divisible Distributions	271
4. Examples for Branching Processes	272
5. Extinction Probabilities in Branching Processes	274
6. Problems for Solution	276
XIII RECURRENT EVENTS. THE RENEWAL EQUATION	278
1. Informal Preparations and Examples	278
2. Definitions	281
3. The Basic Relations	285
4. The Renewal Equation	290
5. Delayed Recurrent Events	293
6. The Number of Occurrences of ε	296
*7. Application to the Theory of Success Runs	299
*8. More General Patterns	303
9. Lack of Memory of Geometric Waiting Times	304
*10. Proof of Theorem 3 of Section 3	306
11. Problems for Solution	308
XIV RANDOM WALK AND RUIN PROBLEMS	311
1. General Orientation	311
2. The Classical Ruin Problem	313
3. Expected Duration of the Game	317
*4. Generating Functions for the Duration of the Game and for the First-Passage Times	318
*5. Explicit Expressions	321
6. Passage to the Limit; Diffusion Processes	323
*7. Random Walks in the Plane and Space	327

CHAPTER	PAGE
8. The Generalized One-Dimensional Random Walk (Sequential Sampling)	330
9. Problems for Solution	334
XV MARKOV CHAINS	338
1. Definition	338
2. Illustrative Examples	340
3. Higher Transition Probabilities	347
4. Closures and Closed Sets	349
5. Classification of States	351
6. Ergodic Properties of Irreducible Chains	356
*7. Periodic Chains	360
8. Transient States	362
9. Application to Card Shuffling	367
10. The General Markov Process	368
*11. Miscellany	373
12. Problems for Solution	376
*XVI ALGEBRAIC TREATMENT OF FINITE MARKOV CHAINS	380
1. General Theory	380
2. Examples	384
3. Random Walk with Reflecting Barriers	388
4. Transient States; Absorption Probabilities	392
5. Application to Recurrence Times	395
XVII THE SIMPLEST TIME-DEPENDENT STOCHASTIC PROCESSES	397
1. General Orientation	397
2. The Poisson Process	400
3. The Pure Birth Process	402
*4. Divergent Birth Processes	404
5. The Birth and Death Process	407
6. Exponential Holding Times	411
7. Waiting Line and Servicing Problems	413
8. The Backward (Retrospective) Equations	421
9. Generalization; The Kolmogorov Equations	423
10. Processes Involving Escapes	428
11. Problems for Solution	434
ANSWERS TO PROBLEMS	437
INDEX	451