

Contents

1 · Sample Spaces	1
1. Random Experiments and Sample Spaces	1
2. Set Notation	2
3. Combinations of Sets	5
4. Venn Diagram	9
5. Events	11
Exercises	13
2 · Combinatorial Probability	18
1. Introduction	18
2. Axioms of Probability	20
3. Elementary Properties of Probability	21
4. Discrete Sample Spaces; Equally Likely Sample Points	22
5. Counting: Permutation and Combinations	25
6. Stirling's Formula	29
7. Further Examples	30
8. Conditional Probability	34
9. Independent Events	39
10. Further Examples	41
Exercises	43
3 · Random Variables	50
1. Introduction	50
2. The Definition of a Random Variable	50
3. Events Defined by Random Variables	53
4. Distribution Functions	57
5. Determining Probabilities from the Distribution Function	62
6. Discrete Random Variables	63
7. Continuous Random Variables	67
Exercises	76

4 · Sets of Random Variables and Random Sequences	83
1. Introduction	83
2. Pairs of Random Variables	83
3. Distribution Functions	86
4. Properties of Joint Distribution Functions	88
5. Determining Probabilities from a Joint Distribution Function	94
6. Density Functions: Discrete Case	99
7. Density Functions: Continuous Case	104
8. Conditional Distributions	110
9. Many Random Variables	119
10. Random Sequences or Discrete Parameter Random Processes	122
11. Random Sequences: Independent Case	124
12. Random Sequences: Markov Dependent Case	125
Exercises	130
 5 · Expectation	136
1. Introduction	136
2. Expectation	136
3. Functions of a Random Variable	139
4. Expectation of a Function of a Random Variable	141
5. Linear Properties of Expectation	143
6. Moments: Mean and Variance	143
7. The Chebyshev Inequality	147
8. Variance of a Linear Combination	148
9. Functions of Several Random Variables; Sums of Random Variables	149
10. Expectation of a Function of Several Random Variables	152
11. Variance of a Sum	153
12. The Moment Generating Function	155
13. Probability Generating Function, z-Transform, and Characteristic Function	159
Exercises	161
 6 · Special Distributions	166
1. Introduction	166
2. The Bernoulli Density Function	166
3. The Binomial Density Function	167
4. The Geometric Density Function	169

CONTENTS	xiii
5. The Poisson Density Function	174
6. The Uniform Density Function	178
7. The Negative Exponential Density Function	180
8. The Gamma Density Function	186
9. The Normal Density Function	189
10. The Cauchy Density Function	194
Exercises	195
7 · Stochastic Processes: Examples	197
1. Introduction	197
2. Stochastic Processes	198
3. Independent Sequences	201
4. Two-State Markov Sequences	203
5. Random Walks	206
6. Gambler's Ruin	208
7. The Poisson Process	210
Exercises	215
8 · Discrete Parameter Markov Processes: The Finite, Irreducible Case	217
1. Introduction	217
2. Some Definitions	217
3. Examples and Preliminary Remarks	220
4. n-step Transition Probabilities	224
5. Examples	226
6. nth-step State Probabilities	228
7. Classification of States	231
8. Examples	231
9. Limiting Distributions	233
10. Examples	235
Exercises	237
9 · Algebraic Methods Useful in the Study of Markov Chains	244
1. Introduction	244
2. Eigenvalues and Eigenvectors	244
3. Examples	247
4. Difference Equations	253
5. Examples of Difference Equations	255
Exercises	258

10 · Nonirreducible or Nonfinite Markov Chains	259
1. Introduction	259
2. Nonirreducible Markov Chains	259
3. Examples	261
4. Infinite Irreducible Markov Chains	263
5. Infinite Nonirreducible Chains	268
Exercises	268
11 · Continuous Parameter Markov Chains	272
1. Introduction	272
2. The Markov property	273
3. Examples	276
4. Properties of Transition Probabilities	277
5. The Rate Matrix and the Kolmogorov Differential Equations	280
6. Examples	283
7. The Birth-and-Death Process	292
8. Algebraic Methods for Solving Kolmogorov Equations	293
Exercises	295
12 · Limiting Distributions of Continuous Parameter Markov Processes	298
1. Introduction	298
2. The Existence of the $\{q_i\}$	299
3. Determination of $\{q_i\}$; the Normal Equations	300
4. The Existence of a Limiting Probability Distribution	301
5. Irreducible Examples	302
6. An Interpretation of the $\{q_i\}$	306
7. Some Nonirreducible Processes	307
Exercises	309
13 · Introduction to Queueing Theory	310
1. Introduction	310
2. Definitions	311
3. The Simple Markovian Queue	313
4. The Multiple-Server Markovian Queue	314
5. A Simple Markov Queue with Bounded Storage	315
6. Independent Interarrival and Service Times	316
7. The M/G/1 Queue. Embedded Chains	317
8. Examples	322
Exercises	325

14 · Further Properties of Stochastic Processes	327
1. Introduction	327
2. Notation and Definitions	327
3. Some General Properties of a Random Process	329
4. Normal Processes; the Wiener Process	334
5. Ergodicity	334
Bibliography	338
Index	341