

CONTENTS

Notation and Symbols					
Chapter 1	Elem	Elements of Linear System Theory 1.1 Introduction, 1			
	1.1				
	1.2	State 1.2.1	Description of Linear Systems, 1 State Description of Nonlinear and Linear Differential Systems, 1		
		1.2.3	Linearization, 2 Examples, 3 State Transformations, 10		
	1.3				
		1.3.1	•		
		1.3.2			
			Diagonalization, 15 The Jordan Form, 19		
	1.4				
			Definitions of Stability, 24		
		1.4.2	•		
		1.4.3*	Stable and Unstable Subspaces for Time-Invariant Linear Systems, 29		
		1.4.4*			
	1.5	Transfe	orm Analysis of Time-Invariant Systems, 33		
		1.5.1	Solution of the State Differential Equation through Laplace Transformation, 33		

* See the Preface for the significance of the marked sections.

21

1

- 1.5.2 Frequency Response, 37
- 1.5.3 Zeroes of Transfer Matrices, 39
- 1.5.4 Interconnections of Linear Systems, 43
- 1.5.5* Root Loci, 51

1.6* Controllability, 53

- 1.6.1* Definition of Controllability, 53
- 1.6.2* Controllability of Linear Time-Invariant Systems, 55
- 1.6.3* The Controllable Subspace, 57
- 1.6.4* Stabilizability, 62
- 1.6.5* Controllability of Time-Varying Linear Systems, 64

1.7* Reconstructibility, 65

- 1.7.1* Definition of Reconstructibility, 65
- 1.7.2* Reconstructibility of Linear Time-Invariant Systems, 67
- The Unreconstructible Subspace, 70 1.7.3*
- 1.7.4* Detectability, 76
- 1.7.5* Reconstructibility of Time-Varying Linear Systems, 78
- 1.8* Duality of Linear Systems, 79
- 1.9* Phase-Variable Canonical Forms, 82
- Vector Stochastic Processes, 85 1.10
 - 1.10.1 Definitions, 85
 - 1.10.2 Power Spectral Density Matrices, 90
 - 1.10.3 The Response of Linear Systems to Stochastic Inputs, 91
 - 1.10.4 Quadratic Expressions, 94

1.11 The Response of Linear Differential Systems to White Noise, 97

- 1.11.1 White Noise, 97
- 1.11.2 Linear Differential Systems Driven by White Noise, 100
- 1.11.3 The Steady-State Variance Matrix for the Time-Invariant Case, 103
- Modeling of Stochastic Processes, 106 1.11.4
- Quadratic Integral Expressions, 108 1.11.5

1.12 *Problems*, 113

		Contents	X1			
Chapter 2	Analysis of Linear Control Systems					
	2.1	Introduction, 119				
-	2.2	 The Formulation of Control Problems, 121 2.2.1 Introduction, 121 2.2.2 The Formulation of Tracking and Regulator Problems, 121 2.2.3 The Formulation of Terminal Control Problems, 127 				
	2.3	Closed-Loop Controllers; The Basic Design Objective, 128				
	2.4	The Stability of Control Systems, 136				
	2.5	 The Steady-State Analysis of the Tracking Properties, 140 2.5.1 The Steady-State Mean Square Tracking Error and Input, 140 2.5.2 The Single-Input Single-Output Case, 144 2.5.3 The Multiinput Multioutput Case, 155 				
	2.6	The Transient Analysis of the Tracking Properties, 165				
	2.7	The Effects of Disturbances in the Single-Input Single-Output Case, 167				
	2.8	The Effects of Observation Noise in the Single-Input Single-Output Case, 174				
	2.9	The Effect of Plant Parameter Uncertainty in the Single-Input Single-Output Case, 178				
	2.10*	The Open-Loop Steady-State Equivalent Control Scheme, 183				
	2.11	Conclusions, 188				
	2.12	Problems, 189				
Chantar 2	Ontin	nal Linean State Feedback Control Systems	103			

Introduction, 193

Feedback, 193

3.2.1

Stability Improvement of Linear Systems by State

Linear State Feedback Control, 193

3.1 3.2

3.2.2*	Conditions for Pole Assignment and
	Stabilization, 198

3.3 The Deterministic Linear Optimal Regulator Problem, 201

- 3.3.1 Introduction, 201
- 3.3.2 Solution of the Regulator Problem, 207
- 3.3.3 Derivation of the Riccati Equation, 216

3.4 Steady-State Solution of the Deterministic Linear Optimal Regulator Problem, 220

- 3.4.1 Introduction and Summary of Main Results, 220
- 3.4.2* Steady-State Properties of Optimal Regulators, 230
- 3.4.3* Steady-State Properties of the Time-Invariant Optimal Regulator, 237
- 3.4.4* Solution of the Time-Invariant Regulator Problem by Diagonalization, 243

3.5 Numerical Solution of the Riccati Equation, 248

- 3.5.1 Direct Integration, 248
- 3.5.2 The Kalman-Englar Method, 249
- 3.5.3* Solution by Diagonalization, 250
- 3.5.4* Solution by the Newton-Raphson Method, 251

3.6 Stochastic Linear Optimal Regulator and Tracking Problems, 253

- 3.6.1 Regulator Problems with Disturbances— The Stochastic Regulator Problem, 253
- 3.6.2 Stochastic Tracking Problems, 257
- 3.6.3 Solution of the Stochastic Linear Optimal Regulator Problem, 259

3.7 Regulators and Tracking Systems with Nonzero Set Points and Constant Disturbances, 270

- 3.7.1 Nonzero Set Points, 270
- 3.7.2* Constant Disturbances, 277

3.8* Asymptotic Properties of Time-Invariant Optimal Control Laws, 281

3.8.1* Asymptotic Behavior of the Optimal Closed-Loop Poles, 281

			•			
		3.8.2*	Asymptotic Properties of the Single-Input Single-Output Nonzero Set Point Regulator, 297			
-		3.8.3*	The Maximally Achievable Accuracy of Regulators and Tracking Systems, 306			
	3.9*	Sensitiv	sitivity of Linear State Feedback Control Systems,			
	3.10	Conclusions, 318				
3.11 <i>Problems</i> , 319						
Chapter 4 Optimal Linear Reconstruction of the State 4.1 Introduction, 328			ar Reconstruction of the State	328		
	4.2		ers, 329			
			Full-Order Observers, 329			
		4.2.2*	Conditions for Pole Assignment and Stabilization of Observers, 334			
		4.2.3*	Reduced-Order Observers, 335			
	4.3	The Optimal Observer, 339				
		4.3.1	A Stochastic Approach to the Observer			
		4.3.2	Problem, 339 The Noneingular Optimal Observer Problem			
		4.3.2	The Nonsingular Optimal Observer Problem with Uncorrelated State Excitation and			
			Observation Noises, 341			
		4.3.3*	<u> </u>			
			with Correlated State Excitation and			
		121*	Observation Noises, 351 The Time-Invariant Singular Optimal			
		4.3.4	Observer Problem, 352			
		4.3.5*	The Colored Noise Observation Problem,			
			356			
		4.3.6*	Innovations, 361			
	4.4*	Regula Observ 4.4.1*	tality of the Optimal Observer and the Optimal tor; Steady-State Properties of the Optimal ter, 364 Introduction, 364 The Duality of the Optimal Regulator and			
		4.4.2*	The Duality of the Optimal Regulator and the Optimal Observer Problem, 364			

		 4.4.3* Steady-State Properties of Observer, 365 4.4.4* Asymptotic Properties of Steady-State Optimal Observer 	Time-Invariant		
	4.5	Conclusions, 373			
	4.6	Problems, 373			
Chapter 5	Opti	Optimal Linear Output Feedback Control Systems			
	5.1	Introduction, 377			
	5.2	The Regulation of Linear Systems Measurements, 378 5.2.1 The Structure of Output Fe Systems, 378 5.2.2* Conditions for Pole Assignr Stabilization of Output Fe Systems, 388	eedback Control		
	5.3	Optimal Linear Regulators with Noisy Measurements, 389 5.3.1 Problem Formulation and S 5.3.2 Evaluation of the Performa Output Feedback Regulator 5.3.3* Proof of the Separation Prin	Solution, 389 ance of Optimal s, 391		
	5.4	Linear Optimal Tracking Systems and Noisy Measurements, 402	with Incomplete		
	5.5	Regulators and Tracking Systems w Points and Constant Disturbances, 5.5.1 Nonzero Set Points, 409 5.5.2* Constant Disturbances, 414			
	5.6*	Sensitivity of Time-Invariant Optimo Feedback Control Systems, 419	al Linear Output		
	5.7*	Linear Optimal Output Feedback Reduced Dimensions, 427 5.7.1* Introduction, 427 5.7.2* Controllers of Reduced Dim 5.7.3* Numerical Determination o trollers of Reduced Dimensi	nensions, 428 f Optimal Con-		

442

	5.8	Conclusions, 436				
	5.9	Problems, 438				
Chapter 6*	Line	ear Optimal Control Theory for Discrete-Time Systems				
	6.1	Introduction, 442				
	6.2	•	of Linear Discrete-Time Systems, 442			
		6.2.1	Introduction, 442			
		6.2.2	State Description of Linear Discrete-Time Systems, 443			
		6.2.3	Interconnections of Discrete-Time and Continuous-Time Systems, 443			
		6.2.4	Solution of State Difference Equations, 452			
		6.2.5	Stability, 454			
		6.2.6	Transform Analysis of Linear Discrete-Time			
			Systems, 455			
		6.2.7	Controllability, 459			
		6.2.8	Reconstructibility, 462			
		6.2.9	Duality, 465			
			Phase-Variable Canonical Forms, 466			
		6.2.11	Discrete-Time Vector Stochastic Processes, 467			
		6.2.12	Linear Discrete-Time Systems Driven by White Noise, 470			
	6.3	Analysi 475	s of Linear Discrete-Time Control Systems,			
		6.3.1	Introduction, 475			
		6.3.2	Discrete-Time Linear Control Systems, 475			
		6.3.3	The Steady-State and the Transient Analysis of the Tracking Properties, 478			
		6.3.4	Further Aspects of Linear Discrete-Time Control System Performance, 487			
	6.4	Optima	l Linear Discrete-Time State Feedback			
		Control	Systems, 488			
		6.4.1	Introduction, 488			
		6.4.2	Stability Improvement by State Feedback, 488			
		6.4.3	The Linear Discrete-Time Optimal Regulator Problem, 490			

6.5

6.6

6.7

6.8

References

Index

6.6.4

bances, 543

Conclusions, 546

Problems, 547

6.4.4	Steady-State Solution of the Discrete-Time Regulator Problem, 495
6.4.5	The Stochastic Discrete-Time Linear Optimal Regulator, 502
6.4.6	Linear Discrete-Time Regulators with Non- zero Set Points and Constant Disturbances, 504
6.4.7	Asymptotic Properties of Time-Invariant Optimal Control Laws, 509
6.4.8	Sensitivity, 520
Optim	al Linear Reconstruction of the State of Linear
Discre	te-Time Systems, 522
6.5.1	Introduction, 522
6.5.2	The Formulation of Linear Discrete-Time Reconstruction Problems, 522
6.5.3	Discrete-Time Observers, 525
6.5.4	Optimal Discrete-Time Linear Observers, 528
6.5.5	Innovations, 533
6.5.6	Duality of the Optimal Observer and Regulator Problems; Steady-State Prop- erties of the Optimal Observer, 533
Optim	al Linear Discrete-Time Output Feedback
Systen	ns, 536
6.6.1	,,,,,,
6.6.2	The Regulation of Systems with Incomplete Measurements, 536
6.6.3	Optimal Linear Discrete-Time Regulators with Incomplete and Noisy Measurements, 539

Nonzero Set Points and Constant Distur-

553

563