## CONTENTS

## PART I

Introduction	to	Scientific	Measurement
Intiounction	w	Determine	Measurement

1.	The Meaning of Measurement	3
	A Definition of Measurement	3 5
	Dimensional Analysis	5
	Testing Hypotheses	6
2.	The Conduct of an Experimental Investigation	7
	Design	7
	Crucial Times During an Investigation	8
3.	The Scientific Report	9
	Functions of the Scientific Report	9
	Components of the Scientific Report	10
4.	Procedure in a Laboratory and the Laboratory Notebook	12
	The Laboratory Notebook	12
5.	Experimental Errors	14
	Random Errors	14
	Systematic Errors	15
	Blunders	16
	Analysis of Errors	16
6.	Rejection of Data: Chauvenet's Criterion and Its Dangers	17
7.	The Philosophy of Sampling and the Definition of Statistical	
	Concepts	19
	Population Parameters	19
	Variation and Distributions	20
	Functions of Statistics	21
	Definitions of Basic Statistical Concepts	21
<b>B.</b>	Discussion of the Analysis of Samples	30
9.	Discussion of Discrete and Continuous Frequency Distributions and	
	Histograms	32

	Normalized Frequency Distribution	32
	Normalized Frequency Histogram	33
	Definitions	35
	Skewness	37
	Sheppard's Correction for Grouping Data	37
10.	Propagation of Error and Least Squares	39
	General Case of Error Propagation	39
	Independent Errors	40
	Graphical Description of Error Propagation	41
	Minimum Variance (Least-Squared Error)	42
	Nonindependent or Correlated Errors	43
	Covariances of Calculated Quantities	45
	Generalization of (10.3)	45
	Error Propagation with Complex Variables	47
DAI	RT II	
	tion to Graphical Techniques and Curve Fitting	
		51
11.	<i></i>	51
	Horizontal Bar Chart	51 51
	Pie Chart Or Area Diagram	51 52
	Volumetric or Solid Diagram	52
12.	Correlation Graphs	53 53
	Silhouette Chart	54
	Horizontal Bar Chart	54
•	Vertical Bar Chart	54
	Line Chart Search for Correlations	55
	Correlation Coefficient	62
12		63
13.		63
	Use of Perspective Projections	63
	Contour Plots	64
14.		71
14.	Straight Line	71
	Calculation of the Least-Squares Straight Line	74
	Fitting Straight-Line Data When Both Variables Have Uncertainties	75
18		76
15.	Log Plots	76
	Semilog Plots	79
	General Use of Logarithmic Scales	87
1.0		90
16.	Fixed Scales	90
		90
	Sliding Scales	91
•	Nomograms	

## PART III Probability

17.	The Meaning of Probability	101
1	Random Phenomena and Random Variables	101
	Probability Distributions and Their Description	102
	Chebychev's Inequality	102
	Derivation of Chebychev's Inequality	10.
	Symmetrical and Asymmetrical Distributions Different Kinds of Brokehility	106
18.	Different Kinds of Probability Some Arithmetic on Combinations and Permutations	107
10.		111
	Arrangements: Permutations and Variations	111
	Combinations The Dispersion The Combinations	112
	The Binomial Theorem	112
	The Laplace Triangle	113
	Remarks on the Factorial Function	115
	Some Necessary Specifications in Combinatorial Analysis	116
	The Multinomial Theorem	118
19.	Event Calculus—The Logic of Probability	121
	Definitions	121
	Conditional Probabilities: Dependence and Independence	125
	Expectation Values—Recursion Relations	136
20.	Joint Probability Distributions and Functions of Random Variables	139
	Joint and Marginal Probability Distributions	139
	Expectation Values	140
	Independence	141
	Covariance	142
	Variance	144
	Calculus of Probability Density Functions (Univariate)	147
	Calculus of Probability Density Functions (Multivariate)	148
21.	Geometrical Probability, Random Numbers, and Monte Carlo	
	Experiments	153
	Buffon's Needle	153
	Bertrand's Paradox	156
	Randomness and Random Numbers Drawn from the Uniform	
	Distribution	157
	Simulation of Probability Problems: Monte Carlo Experiments	158
	Operations with Random Numbers	162
	Sums of Random Numbers	162
	Random Numbers Drawn from an Arbitrary Distribution	167
	Correlations	169
		102
		•
PART IV		
	obability Distributions and Applications	
		101
22.	The Binomial Distribution	181
	Definitions  Descriptions	181
	Reproductive Property of the Binomial Distribution	183
	Probability of a Range of Values	183

	Symmetry and Asymmetry	184
	Expectation Value of the Binomial Distribution	185
	Variance of the Binomial Distribution	186
	The Expectation Value for the Number of Trials Required	
	for a Specified Number of Successes	186
	The Mode of the Binomial Distribution	187
	Rare Events	189
	Inverse Probability	189
	Law of Large Numbers	190
	The Frequency Definition of Probability	191
	The Law of Large Numbers and the Sample Mean	192
23.	The Hypergeometric Distribution	193
	Definition of the Probability	193
	Expectation Value and Variance	194
	Binomial Approximation to the Hypergeometric Distribution	199
	Inverse Probability	200
24.	The Poisson Distribution	202
	Exact Model	202
	Poisson Approximation to the Binomial Distribution	207
	Expectation Value for the Poisson Distribution	212
	Variance for the Poisson Distribution	212
	Poisson Approximation to the Hypergeometric Distribution	212
	Reproductive Property of the Poisson Distribution	213
	Radioactive Decay and the Exponential Decay Distribution	213
	The Binomial Distribution of Poisson (BDOP)	214
	Interval Distribution	216
	Inverse Probability	216
	Cumulative Poisson—Distribution Function	219
25.	The Gaussian or Normal Distribution	223
	Derivation of the Gaussian Distribution from Certain	
	Assumptions	223
	Relation of the Mean Deviation to the Standard Deviation	226
	Derivation of Gaussian Distribution from the Binomial Distribution	226
	Derivation of Gaussian Distribution from the Poisson Distribution	227
	Inverse Probability	228
	Some Properties of the Normal Distribution	229
	Normal Deviate Test for the Difference of Two Sample Means	235
	Normal Approximation to the Binomial Distribution	237
	The Central Limit Theorem (Normal Convergence Theorem)	244
26.	The Chi-Square Distribution	254
	Chi-Square and Minimization	254
	Probability Density Functions for n Independent Degrees	250
	of Freedom	259
	Mean, Mode, and Variance of the Chi-Square Distribution	261
	Computations with the Chi-Square Distribution	263
	Approximation to the Chi-Square Distribution	264
	The Sample Variance	269
27.	Student's t Distribution	274
	Definition of $t$ and its p.d.f.	274
	Cauchy Distribution	275
	Applications of Student's t Distribution	279

28.	Miscellaneous Other Probability Distributions and Examples	283
	The Negative Binomial Distribution	283
	The Multinomial Distribution	284
	The Exponential Distribution	284
	The Weibull Distribution	284
	The Log-Normal Distribution	285
	The F-distribution	285
	Folded Distributions	286
	Folded Normal Distribution	287
	Truncated Distributions	287
	Truncated Normal Distribution	287 288
	The Bivariate Normal Distribution  Multivariate Normal Distribution	290
	Multivariate Normai Distribution	270
PART V		
Statistic	al Inference	
29.	Estimation	293
	Confidence Intervals	294
	Estimation of a Population Mean for a Large, Homogeneous	
	Population	295
	Estimation of Population Mean for a Finite Population	296
	Stratified Sampling: Estimation of a Population Mean where the	207
	Population is Large and Partitioned into Strata	297
	Estimation of a Probability (Binomial)	307
20	Estimation of a Population Proportion	309 311
30.	Estimation and the Method of Maximum Likelihood	311
	Likelihood Estimators	317
	General Properties of Estimators  Nonanalytical Solution of the Likelihood Equation	326
	Asymptotic Properties of the Likelihood Function	329
	Finite Data	330
	Error Expected Prior to a Measurement	331
	Inefficient Statistics	332
	Graphical Methods: the Score Function	333
31.	Hypothesis Testing and Significance	340
<b>31.</b>	Kinds of Hypotheses	340
	Consistency and Proof	340
	Two Kinds of Error and the Cost of Being Wrong	341
	Concepts in Hypothesis Testing	342
	The Neyman-Pearson Theorem	350
	The Likelihood Ratio	352
	The Generalized Likelihood Ratio	352
	Large-Sample Properties of the Likelihood Ratio	354
	The Generalized $\chi^2$ Test for Goodness-of-Fit	356
	Use of the $\chi^2$ Test for Goodness-of-Fit	352
32.	Chi-Square Minimization Methods	351
	Review of $\chi^2$	359
	One-Parameter $\chi^2$ : $\mathbf{c} = c_1$	361
	Multiparameter $y^2$ : $\mathbf{c} = (c_1, \dots, c_r)$	364

	33.	Least-Squares Methods; Curve Fitting	387
		General Formulation	387
		Linear Case	388
		Goodness-of-Fit	397
		Linear Least Squares with Linear Constraints	397
		Nonlinear Least Squares	399
ADDE	ENDIC	Ee	
AFFE			401
	A.	Review of Notation and Some Elementary Mathematics	401
		Maxima and Minima of Functions	405
	В.	Matrices, Determinants, and Linear Equations	407
	I.	Units and Standards of Weights and Measures	419
	II.	Dimensional Analysis	431
	Ш.	Some Comments on the Factorial, Gamma, and Error Functions	434
		Stirling's Formula	436
		Gamma and Beta Functions	437
	IV.	Eight Hundred Uniformly Distributed Random Numbers and Eight	
		Hundred Random Normal Deviates	439
	V.	Tables of the Negative Exponential $e^{-x}$	449
	VI.	Tables of the Gaussian (Normal) Distribution	457
	VII.	Tables and Graphs of the Chi-Square Distribution	465
	VIII.		483
	Guid	e for Further Reading and Bibliography	449
	Inde	_	505