

Prefe	page	xxiii
	CHAPTER 1. INTRODUCTION	
1.1	The composition of the ionosphere	I
1.2	Plane waves and spherical waves. The curvature of the earth	I
1.3	Effect of collisions and of the earth's magnetic field	2
1.4	Relation to other kinds of wave-propagation	2
1.5	The variation of electron density with height. The Chapman layer	3
1.6	Approximations to the electron density profile	5
1.7	The variation of collision-frequency with height	6
1.8	The structure of the ionosphere	7
1.9	Horizontal variations and irregularities	10
	CHAPTER 2. THE BASIC EQUATIONS	
2.I	Units	II
2.2	Harmonic waves and complex quantities	12
2.3	Definitions of electric intensity <b>E</b> and magnetic intensity <b>H</b>	13
2.4	The current density $J$ and electric polarisation $P$	14
2.5	The electric displacement ${\bf D}$ and magnetic induction ${\bf B}$	15
2.6	Maxwell's equations	16
2.7	Cartesian coordinate system	16
2.8	Progressive plane waves	17
2.9	Plane waves in free space	19
2.10	The notation ${\mathcal H}$ and ${\mathbf H}$	20
2.11	The energy stored in a radio wave in the ionosphere	20
2.12	The flow of energy. Poynting's theorem	21
2.13	The Poynting vector	22

	CHAPTER 3. THE CONSTITUTIVE RELATION	NS
3.1	Introduction	page 24
3.2	Free, undamped electrons	24
3.3	Electron collisions. Damping of the motion	25
3.4	Effect of the earth's magnetic field on motion of electrons	26
3.5	The effect of the magnetic field of the wave on the motion of electrons	of 28
3.6	The susceptibility matrix	29
3.7	The Lorentz polarisation term	30
3.8	The effect of small irregularities in the ionosphere	31
3.9	The effect of heavy ions	31
3.10	The energy stored in a radio wave in the ionosphere (continued)	33
3.11	The principal axes	35
	CHAPTER 4. PROPAGATION IN A HOMOGENEOUS ISOTROPIC MEDIUM	
<b>4.1</b>	Definition of the refractive index	38
4.2	The Maxwell equation derived from Faraday's law	38
4.3	Isotropic medium without damping	39
4.4	Isotropic medium with collision damping	40
4.5	The physical interpretation of a complex refractive index	41
4.6	Evanescent waves	41
4.7	Inhomogeneous plane waves	42
4.8	Energy flow in inhomogeneous plane waves	44
4.9	The case when $X = 1$ , $n = 0$	45
	HAPTER 5. PROPAGATION IN A HOMOGENINISOTROPIC MEDIUM. MAGNETOIONIC TH	
5.1	Introduction	47
5.2	The wave-polarisation	47
5-3	The polarisation equation	48

	CONTENTS		ix
5-4	Properties of the polarisation equation	page	49
5.5	Alternative measure of the polarisation. Axis ratio and tiltangle		51
5.6	The Appleton-Hartree formula for the refractive index		52
5.7	The longitudinal component of the electric field		53
5.8	The flow of energy for a wave in a magnetoionic medium		54
5.9	The effect of heavy ions on polarisation and refractive inde	x	56

### CHAPTER 6. PROPERTIES OF THE APPLETON-HARTREE FORMULA

57

Examples

0.1	General properties. Zeros and infinity of the refractive index	59
6.2	Collisions neglected	60
6.3	Frequency above the gyro-frequency	60
6.4	Longitudinal propagation when $Y < I$	60
6.5	Transverse propagation when $Y < 1$	61
6.6	Intermediate inclination of the field when $Y < I$	62
6.7	Frequency below the gyro-frequency	64
6.8	Longitudinal propagation when $Y > 1$	64
6.9	Transverse propagation when $Y > 1$	65
6.10	Intermediate inclination of the field when $Y > 1$	65
6.11	Effect of collisions included	66
6.12	The critical collision-frequency	67
6.13	Longitudinal propagation when collisions are included	69
6.14	Transverse propagation when collisions are included	70
6.15	Intermediate inclination of the field	72
6.16	The 'quasi-longitudinal' approximation	76
6.17	The 'quasi-transverse' approximation	77
6.18	The effect of heavy ions	78

BRW

# CHAPTER 7. DEFINITION OF THE REFLECTION AND TRANSMISSION COEFFICIENTS

7.1	Introduction	page 85
7.2	The reference-level for reflection coefficients	85
7.3	The reference-level for transmission coefficients	87
7.4	The four reflection coefficients and the four transmission coefficients	88
7.5	The sign convention	88
7.6	The reflection coefficient matrix	90
7.7	Alternative forms of the reflection coefficients	90
7.8	Spherical waves	91
	Examples	94
СН	APTER 8. REFLECTION AT A SHARP BOUND	ARY
8.1	Introduction	96
8.2	The boundary conditions .	96
8.3	Snell's law	97
8.4	Derivation of the Fresnel formulae for isotropic media	98
8.5	General properties of the Fresnel formulae	100
8.6	The Fresnel formulae when the electric vector is in the plane of incidence	101
8.7	The Fresnel formulae when the electric vector is horizontal	105
8.8	Reflection when $X = 1$ , $Z = 0$ , $n = 0$	106
8.9	Normal incidence	108
8.10	Homogeneous ionosphere with parallel boundaries	108
8.11	Normal incidence on a parallel-sided slab	112
8.12	Reflection at normal incidence when the earth's magnetic field is allowed for	114
8.13	Earth's magnetic field horizontal. Normal incidence	115
8.14	Earth's magnetic field vertical. Normal incidence	116
-	Reflection when the earth's magnetic field is included.  Approximate formulae for oblique incidence	116

	CONTENTS	хi
8.16	The validity of the approximations pa	ge 119
8.17	Reflection at oblique incidence. The Booker quartic	120
8.18	Some properties of the Booker quartic	123
8.19	Reflection at oblique incidence for north-south or	
	south-north propagation	124
8.20	Reflection at oblique incidence in the general case	126
	Examples	127
	CHAPTER 9. SLOWLY VARYING MEDIUM. THE W.K.B. SOLUTIONS	
9.1	Introduction	128
9.2	The differential equations	128
9.3	The phase memory concept	130
9.4	Loss-free medium. Constancy of energy-flow	131
9.5	Derivation of the W.K.B. solution	131
9.6	Condition for the validity of the W.K.B. solutions	133
9.7	Properties of the W.K.B. solutions	134
9.8	The reflection coefficient	136
9.9	Coupling between upgoing and downgoing waves	137
9.10	Extension to oblique incidence	138
9.11	The differential equations for oblique incidence	140
9.12	The W.K.B. solutions for horizontal polarisation at oblique	
	incidence	140
9.13	The W.K.B. solutions at oblique incidence when the electric	
	vector is parallel to the plane of incidence	142
	The effect of including the earth's magnetic field	143 144
9.15	Ray theory and 'full wave' theory	*44
	CHAPTER 10. RAY THEORY FOR VERTICAL INCIDENCE WHEN THE EARTH'S MAGNETI FIELD IS NEGLECTED	
10.1	The use of pulses	146
10.2	The group velocity	147
10.3	The equivalent height of reflection $h'(f)$	149
		b-2

W	Ť	Ť.

10.4	The 'true height' and the 'phase height' page	150
10.5	The equivalent height of reflection for a linear profile of electron density	150
10.6	The equivalent height of reflection for an exponential variation of electron density	151
10.7	Equivalent height for a parabolic profile of electron density	152
10.8	Equivalent height for the 'sech2' profile of electron density	156
10.9	Two separate parabolic layers	157
10.10	The effect of a 'ledge' in the electron density profile	158
10.11	The calculation of electron density $N(z)$ , from $h'(f)$ data	160
10.12	Solution when $N(z)$ is monotonic	161
10.13	Partial solution when $N(z)$ is not monotonic	163
10.14	The shape of a pulse of radio waves	166
10.15	The effect of electron collisions on group refractive index	170
10.16	The effect of collisions on equivalent height $h'(f)$ and phase height $h(f)$	171
10.17	Relation between equivalent height, phase height and absorption	172
	Examples	174
	CHAPTER 11. RAY THEORY FOR OBLIQUE INCIDENCE WHEN THE EARTH'S MAGNETIC FIELD IS NEGLECTED	
II.I	Introduction. The ray path	175
11.2	Wave-packets	177
11.3	The equation for the ray when the earth's magnetic field is neglected	178
11.4	The ray path for a linear gradient of electron density	179
11.5	The ray path for exponential variation of electron density	180
11.6	The ray path for a parabolic profile of electron density	182
11.7	The skip distance	183

	CONTENTS	xiii
8.11	The equivalent path P' at oblique incidence page	185
11.9	Breit and Tuve's theorem. Martyn's theorem for equivalent path	186
11.10	The equivalent path at oblique incidence for a linear gradient of electron density	188
11.11	The equivalent path at oblique incidence for a parabolic profile of electron density	188
11.12	The dependence of signal on frequency near the maximum usable frequency	190
11.13	The prediction of maximum usable frequencies. Appleton and Beynon's method	191
11.14	The curvature of the earth	192
11.15	The prediction of maximum usable frequencies. Newbern Smith's method	194
11.16	The absorption of radio waves. Martyn's theorem for absorption	195
11.17	The effect of electron collisions on equivalent path	197
	Examples	197
	CHAPTER 12. RAY THEORY FOR VERTICAL INCIDENCE WHEN THE EARTH'S MAGNETIC FIELD IS INCLUDED	
12.1	Introduction	199
12.2	Magnetoionic 'splitting'	199
12.3	The group refractive index—collisions neglected	200
12.4	The effect of collisions on the group refractive index	204
12.5	The equivalent height of reflection $h'(f)$ —collisions neglected	205
12.6	The $h'(f)$ curves when collisions are neglected	206
12.7	The penetration-frequencies for the ordinary and extraordinary waves	208
12.8	The equivalent height for a parabolic layer	209
12.9	Two separate parabolic layers	210

xiv	CONTENTS	
12.10	The effect of a 'ledge' in the electron density profile	age 212
12.11	The effect of collisions on equivalent height $h'(f)$	212
12.12	The polarisation of waves in a wave-packet	214
12.13	The calculation of electron density $N(z)$ from $h'(f)$	215
12.14	Example of the use of the method	218
12.15	Other versions of the foregoing method	221
12.16	Failure of the method when $N(z)$ is not monotonic	222
12.17	The use of $h'_x(f)$ for the extraordinary ray	223
	Example	224
	CHAPTER 13. RAY THEORY FOR OBLIQUE INCIDENCE WHEN THE EARTH'S MAGNETI	
	FIELD IS INCLUDED	
13.1	Introduction	225
•	The variable q	225
13.3	Derivation of the Booker quartic	226
13.4	The transition to a continuous medium	228
13.5	The path of a wave-packet	229
13.6	The reversibility of the path	230
13.7	The reflection of a wave-packet	230
13.8	A simple example of ray paths at oblique incidence	231
13.9	Further properties of the Booker quartic	233
13.10	The Booker quartic for east-west and west-east propagation	236
13.11	The Booker quartic for north-south and south-north propagation	238
13.12	The Booker quartic in the general case when collisions are neglected	e 244

246

248

250

13.13 Lateral deviation at vertical incidence

13.15 Lateral deviation in the general case

west or west to east

13.14 Lateral deviation for propagation from (magnetic) east to

	CONTENTS	xv	
13.16	Calculation of attenuation, using the Booker quartic page	250	
13.17	The 'refractive index' surface in a homogeneous medium	252	
13.18	The direction of the ray	253	
13.19	The ray velocity and the ray surface	255	
13.20	Whistlers	256	
13.21	Determination of ray direction by Poeverlein's construction	258	
13.22	Propagation in the magnetic meridian. The 'Spitze'	260	
13.23	The refractive index surfaces for the extraordinary ray when $Y < I$	262	
13.24	The refractive index surfaces for the extraordinary ray when $Y > 1$	266	
13.25	The second refractive index surface for the ordinary ray	- (0	
	when $Y > 1$	268	
	Examples	270	
CHAPTER 14. THE GENERAL PROBLEM OF RAY TRACING			
14.1	Introduction	271	
14.2	Equations of the refractive index surface and the ray surface	272	
14.3	The Eikonal function	274	
14.4	The canonical equations for a ray, and the generalisation of Snell's law	276	
14.5	Other relations between the equations for the ray surface and the refractive index surface	278	
14.6	Fermat's principle	279	
14.7	Equivalent path and absorption	279	
14.8	The problem of finding the maximum usable frequency	281	
	Examples	282	
CHAPTER 15. THE AIRY INTEGRAL FUNCTION, AND THE STOKES PHENOMENON			
15.1	Introduction	283	
15.2	Linear gradient of electron density associated with an isolated zero of $q$	283	

•

-	-	
•	v	
~	<b>V</b> I	

15.3	The differential equation for horizontal polarisation and oblique incidence	page	285
15.4	·	. 0	286
15.5	Qualitative discussion of the solutions of the Stokes equation		287
15.6	Solutions of the Stokes equation expressed as contour integrals	-	288
15.7	Solutions of the Stokes equation expressed as Bessel functions		291
15.8	Tables of the Airy integral functions		-y- 291
	The W.K.B. solutions of the Stokes equation		-9- 292
	The Stokes phenomenon of the 'discontinuity of the constants'		
15.11	Stokes lines and anti-Stokes lines		292
	The Stokes diagram		293 204
	Definition of the Stokes constant		294 295
	Furry's derivation of the Stokes constants for the Stokes equation		-93 296
15.15	Asymptotic approximations obtained from the contour integrals		297
15.16	Summary of some important properties of complex variables		297
15.17	Integration by the method of steepest descents		300
15.18	Application of the method of steepest descents to solution of the Stokes equation	ıs	302
	Integration by the method of stationary phase		302 307
	Method of stationary phase applied to the Airy integral function		
15 21	Asymptotic expansions		309
		Ì	310
	The chains of a fundamental automatic approximations	:	310
15.23	The choice of a fundamental system of solutions of the Stokes equation	,	312
15.24	Connection formulae, or circuit relations	,	313
15.25	The intensity of light near a caustic	,	313

C	0	N	т	E	N	T	c
u	v	1.4		L	N		2

xvii

# CHAPTER 16. LINEAR GRADIENT OF ELECTRON DENSITY

16.1	Introduction	<i>page</i> 319
16.2	Purely linear profile. Electron collisions neglected	319
16.3	Application to a slowly varying profile	322
16.4	The effect of electron collisions. The height $z$ as a complevariable	ex 326
16.5	Constant collision-frequency. Purely linear profile of electron density	327
16.6	The slowly varying profile when collisions are included. Derivation of the phase integral formula	329
16.7	Discussion of the phase integral formula	331
16.8	Effect of curvature of the electron density profile	333
16.9	Reflection at a discontinuity of gradient	334
16.10	Linear gradient between two homogeneous regions	336
16.11	Symmetrical ionosphere with double linear profile	340
16.12	The differential equation for oblique incidence applicable when the electric vector is parallel to the plane of incidence	
16.13	The behaviour of the fields near a zero of the refractive index for 'vertical' polarisation at oblique incidence	346
16.14	The generation of harmonics in the ionosphere	347
16.15	The phase integral formula for 'vertical' polarisation at oblique incidence	348
16.16	Asymptotic approximations for the solutions of the differential equation for 'vertical' polarisation	349
16.17	Application of the phase integral formula	350
	CHAPTER 17. VARIOUS ELECTRON DENSITE PROFILES WHEN THE EARTH'S MAGNETING FIELD IS NEGLECTED	
17.1	Introduction	353
17.2	Exponential profile. Constant collision-frequency	354
17.3	The phase integral formula applied to the exponential lay-	er 357

xviii	CONTENTS	
17.4	The parabolic layer	age 358
17.5	Partial penetration and reflection	363
17.6	The equivalent height of reflection for a parabolic layer	365
17.7	Electron density with square law increase	366
17.8	The sinusoidal layer	368
17.9	Circuit relations. Introduction to Epstein's theory	369
17.10	The hypergeometric differential equation	370
17.11	The circuit relations for the hypergeometric function	372
17.12	Application to the wave-equation	375
17.13	The reflection and transmission coefficients of an Epstein layer	377
17.14	Epstein profiles	377 378
	Ionosphere with gradual boundary	380
_	The 'sech2' profile	380
-	Fixed electron density and varying collision-frequency	383
•		5 5
C	CHAPTER 18. ANISOTROPIC MEDIA. COUPL	
18.1	WAVE-EQUATIONS AND W.K.B. SOLUTIONS Introduction	_
	The differential equations	385 285
	The four characteristic waves	385 387
18.4	Matrix form of the equations	389 389
18.5	The differential equations for vertical incidence	391
18.6	The W.K.B. solution for vertical incidence on a loss-free	37-
	medium	392
18.7	Introduction to W.K.B. solutions in the general case	394
18.8	Introduction to coupled wave-equations	394
18.9	Försterling's coupled equations for vertical incidence	396
18.10	Coupled equations in the general case, in matrix form	398
<b>TQ TT</b>	Expressions for the elements of S S-1 and S-1S'	200

	CONTENTS		xix
18.12	The W.K.B. solutions in the general case	page	401
18.13	The first-order coupled equations for vertical incidence		402
18.14	The W.K.B. solutions for vertical incidence		405
18.15	The first-order equations in other special cases		406
18.16	Second-order coupled equations		408
18.17	Condition for the validity of the W.K.B. solutions		410
	Example		<b>41</b> 1
	CHAPTER 19. APPLICATIONS OF COUPLE WAVE-EQUATIONS	D	
19.1	Introduction		412
19.2	Properties of the coupling parameter $\psi$		412
19.3	Behaviour of the coefficients near a coupling point		4 <sup>1</sup> 7
19.4	Properties of the coupled differential equations near a reflection point and near a coupling point		418
19.5	The use of successive approximations		421
19.6	The phase integral formula for coupling		423
19.7	The Z-trace		424
19.8	The method of 'variation of parameters'		426
19.9	The 'coupling echo'		428
19.10	The transition through critical coupling		429
19.11	Introduction to limiting polarisation		432
19.12	The free space below the ionosphere		433
19.13	The differential equation for the study of limiting polarisation		434
	Examples		436
	TIADTED 40 THE DILACE INTECDAL METU	IOD	
	CHAPTER 20. THE PHASE INTEGRAL METE Introduction	עטי	427
20.1	The Riemann surface for the refractive index		437 438
20.2			
	The linear electron density profile		440
20.4	The parabolic electron density profile		443

XX	CONTENTS	
20.5	A further example of the method	<i>page</i> 446
20.6	Coupling branch points and their Stokes lines and anti-Stokes lines	450
20.7	The phase integral method for coupling	452
20.8	Further discussion of the transition through critical coupling (continued from § 19.10)	455
	Example	457
СН	APTER 21. FULL WAVE SOLUTIONS WHEN EARTH'S MAGNETIC FIELD IS INCLUDE:	
21.1	Introduction	458
21.2	The differential equations	458
21.3	Vertical incidence and vertical magnetic field	459
21.4	Exponential profile of electron density. Constant collision-frequency	460
21.5	Exponential profile (continued). Incident wave linearly polarised	462
21.6	Other electron density profiles for vertical field and vertical incidence	cal 464
21.7	Vertical magnetic field and oblique incidence. Introduction to Heading and Whipple's method	464
21.8	Regions O, I and I (a)	465
21.9	Reflection and transmission coefficients of region I	467
21.10	Regions II and II (a)	468
21.11	The reflection coefficients of region II	469
21.12	The combined effect of regions I and II	471
21.13	The effect of an infinity in the refractive index	472
21.14	Isolated infinity of refractive index	474
21.15	Refractive index having infinity and zero	476

21.16 The apparent loss of energy near an infinity of refractive

479 481

index

Example

### CHAPTER 22. NUMERICAL METHODS FOR FINDING REFLECTION COEFFICIENTS

21.1	Introduction	page 482	
22.2	Methods of integrating differential equations	483	
22.3	The size of the step	484	
22.4	The choice of dependent variable	484	
22.5	The three parts of the calculation of reflection coefficien	ts 486	
22.6	The starting solutions at a great height	487	
22.7	Calculation of the components of the reflection coefficient	nt 489	
22.8	The wave-admittance in an isotropic ionosphere	491	
22.9	The wave-admittance matrix A for an anisotropic	400	
22.10	ionosphere The starting value of A	493	
	Relation between the admittance matrix <b>A</b> and the	495	
22.11	reflection coefficient matrix R	496	
22.12	The differential equation for A	498	
22.13	Symmetry properties of the differential equations	499	
22.14	Equivalent height of reflection	499	
	Example	501	
	CHAPTER 23. RECIPROCITY		
23.1	Introduction	502	
23.2	Aerials	502	
23.3	Goubau's reciprocity theorem	505	
23.4	One magnetoionic component	506	
23.5	Reciprocity with full wave solutions	508	
Apper	ndix. The Stokes constant for the differential equation		
	(16.98) for 'vertical' polarisation	510	
Biblio	graphy	512	
Index of definitions of the more important symbols 52			
Subject and name index 53			