

# CONTENTS

## *Chapter I*

Basic Principles .....	1
1. Introduction .....	1
2. Probabilities and Certainties. Ordinary and Generalized Functions.....	3
3. Some Properties of Generalized Functions and Further Examples.....	7
4. Phase Space and the Liouville Equation.....	12
5. An Example: Thermal Equilibrium of a Monatomic Ideal Gas.....	16
6. The Problem of Nonequilibrium States. The Boltzmann Equation.....	22
7. Details of the Collision Operator.....	30
References .....	38

## *Chapter II*

Basic Properties .....	39
1. Elementary Manipulations of the Collision Operator. Collision Invariants .....	39
2. Solution of the Equation $Q(f,f) = 0$ .....	43
3. Connection of the Microscopic Description with Macroscopic Gas Dynamics.....	44
4. Boundary Conditions .....	51
5. The $H$ -Theorem .....	55
6. Equilibrium States and Maxwellian Distributions.....	61
References .....	64

## *Chapter III*

The Linearized Collision Operator .....	66
1. General Comments on Perturbation Methods for the Boltzmann Equation.....	66
2. Basic Properties of the Linearized Collision Operator .....	68
3. Power-Law Potentials and Angular Cutoff.....	73

4. Potentials with a Strictly Finite Range . . . . .	77
5. The Case of Maxwell's Molecules . . . . .	80
References . . . . .	85
 <i>Chapter IV</i>	
Model Equations . . . . .	87
1. Guessing Models: The Nonlinear Bhatnagar, Gross, and Krook Model and Generalizations . . . . .	87
2. Deducing Models: The Gross and Jackson Procedure and Generalizations . . . . .	91
3. Models with Velocity-Dependent Collision Frequency . . . . .	94
4. Models for the Boundary Conditions . . . . .	95
References . . . . .	100
 <i>Chapter V</i>	
The Hilbert and Chapman-Enskog Theories . . . . .	101
1. A Bridge between the Microscopic and Macroscopic Descriptions . . . . .	101
2. The Hilbert Expansion . . . . .	102
3. The Chapman-Enskog Expansion . . . . .	107
4. Advantages and Disadvantages of the Hilbert and Chapman-Enskog Expansions . . . . .	114
5. The Problems of Initial Data, Boundary Conditions, and Shock Layers . . . . .	118
6. Kinetic Models <i>versus</i> Chapman-Enskog Theory . . . . .	123
References . . . . .	124
 <i>Chapter VI</i>	
The Linearized Boltzmann Equation . . . . .	126
1. General Considerations . . . . .	126
2. The Free-Streaming Operator . . . . .	132
3. The Integral Version of the Boltzmann Equation and its Properties . . . . .	136
4. Existence and Uniqueness of the Solution for Linearized and Weakly Nonlinear Boundary Value Problems . . . . .	140
5. Convergence of the Solutions of Kinetic Models . . . . .	141
6. Unbounded Regions and External Flows . . . . .	143
7. Influence of Various Spectra . . . . .	147
8. The Linearized Boltzmann Equation and the Chapman-Enskog Theory . . . . .	151
References . . . . .	154

*Chapter VII*

Analytical Methods of Solution . . . . .	155
1. Introduction . . . . .	155
2. Splitting of the One-Dimensional BGK Equation . . . . .	156
3. Elementary Solutions of the Shear Flow Equation . . . . .	158
4. Application of the General Method to the Kramers Problem	162
5. Application to the Flow between Parallel Plates . . . . .	168
6. Elementary Solutions for Time-Dependent Shear Flows . .	174
7. Analytical Solutions of Specific Problems . . . . .	178
8. More General Models . . . . .	183
9. Some Special Cases . . . . .	187
10. Unsteady Solutions of Kinetic Models with Velocity-Dependent Collision Frequency . . . . .	191
11. Two-Dimensional and Three-Dimensional Problems . . . . .	195
12. Connections with the Chapman-Enskog Method . . . . .	196
13. Sound Propagation and Light Scattering in Monatomic Gases . . . . .	197
References . . . . .	198

*Chapter VIII*

Other Methods of Solution . . . . .	201
1. Introduction . . . . .	201
2. Moment Methods . . . . .	202
3. The Integral Equation Approach . . . . .	203
4. The Variational Principle . . . . .	205
5. Examples of Specific Problems . . . . .	208
6. Concluding Remarks . . . . .	218
References . . . . .	220
Author Index . . . . .	222
Subject Index . . . . .	224