Contents

PAGE

Preface to second edition v

Preface to first edition vii

0. The Many-Body Problem for Everybody 1

0.0 What the many-body problem is about 1
0.1 Simple example of non-interacting fictitious bodies . . . 3
0.2 Quasi particles and quasi horses 4
0.3 Collective excitations 10

1. Feynman Diagrams, or how to Solve the Many-Body Problem by means of Pictures 12

1.1 Propagators—the heroes of the many-body problem . . . 12
1.2 Calculating propagators by Feynman diagrams: the drunken man propagator 13
1.3 Propagator for single electron moving through a metal . . . 17
1.4 Single-particle propagator for system of many interacting particles . 17
1.5 The two-particle propagator and the particle–hole propagator . 22
1.6 The no-particle propagator ('vacuum amplitude'). 23

2. Classical Quasi Particles and the Pinball Propagator 25

2.1 Physical picture of quasi particle 25
2.2 The classical quasi particle propagator 26
2.3 Calculation of the propagator by means of diagrams . . . 28

3. Quantum Quasi Particles and the Quantum Pinball Propagator 37

3.1 The quantum mechanical propagator 37
3.2 The quantum pinball game 43
3.3 Disappearance of disagreeable divergences 51
3.4 Where the diagram expansion of the propagator really comes from . 52
3.5 Energy and lifetime of an electron in an impure metal . . . 54

4. Quasi Particles in Fermi Systems 64

4.1 Propagator method in many-body systems 64
4.2 Non-interacting Fermi system in external potential; particle–hole picture 65
4.3 A primer of occupation number formalism (second quantization) . 67
4.4 Propagator for non-interacting Fermi system in external perturbing potential 72
4.5 Interacting Fermi system 78
4.6 The 'quasi-physical' nature of Feynman diagrams 87
4.7 Hartree and Hartree–Fock quasi particles 89
4.8 Hartree–Fock quasi particles in nuclear matter 92
4.9 Quasi particles in the electron gas, and the random phase approximation 94

PAGE

5. Ground State Energy and the Vacuum Amplitude or 'No-particle Propagator' 101

5.1 Meaning of the vacuum amplitude 101
5.2 The pinball machine vacuum amplitude 103
5.3 Quantum vacuum amplitude for one-particle system . . . 105
5.4 Linked cluster theorem for one-particle system 109
5.5 Finding the ground state energy in one-particle system . . . 111
5.6 The many-body case 115

6. Bird's-Eye View of Diagram Methods in the Many-Body Problem 118

7. Occupation Number Formalism (Second Quantization) 123

7.1 The advantages of occupation number formalism 123
7.2 Many-body wave function in occupation number formalism . . 124
7.3 Operators in occupation number formalism 128
7.4 Hamiltonian and Schrödinger equation in occupation number formalism 132
7.5 Particle–hole formalism 136
7.6 Occupation number formalism based on single-particle position eigenstates 138
7.7 Bosons 140

8. More about Quasi Particles 142

8.1 Introduction 142
8.2 A soluble fermion system: the pure Hartree model . . . 143
8.3 Crude calculation of quasi particle lifetime 149
8.4 General form of quasi particle propagator 151

9. The Single-Particle Propagator Re-visited 154

9.1 Second quantization and the propagator 154
9.2 Mathematical expression for the single-particle Green's function propagator 154
9.3 Spectral density function 158
9.4 Derivation of the propagator expansion in the many-body case . 159
9.5 Topology of diagrams 160
9.6 Diagram rules for single-particle propagator 165
9.7 Modified propagator formalism using chemical potential, μ . . 168
9.8 Beyond Hartree–Fock: the single pair-bubble approximation . . 170

10. Dyson's Equation, Renormalization, RPA and Ladder Approximations 177

10.1 General types of partial sums 177
10.2 Dyson's equation 178
10.3 Quasi particles in low-density Fermi system (ladder approximation) 183
10.4 Quasi particles in high-density electron gas (random phase approximation) 185
10.5 The general 'dressed' or 'effective' interaction 192
10.6 The scattering amplitude 196
10.7 Evaluation of the pair bubble; Friedel oscillations . . . 197

PAGE

11. Self-Consistent Renormalization and the Existence of the Fermi Surface 203

11.1 Dressed particle and hole lines, or 'clothed skeletons' . . . 203
11.2 Existence of quasi particles when the perturbation expansion is valid 205
11.3 Existence of the Fermi surface in an interacting system . . . 209
11.4 Dressed vertices 211

12. Ground State Energy of Electron Gas and Nuclear Matter 214

12.1 Review 214
12.2 Diagrams for the ground state energy 214
12.3 Ground state energy of high-density electron gas: theory of Gell-Mann and Brueckner 217
12.4 Brief view of Brueckner theory of nuclear matter 222

13. Collective Excitations and the Two-Particle Propagator 227

13.1 Introduction 227
13.2 The two-particle Green's function propagator 228
13.3 Polarization ('density fluctuation') propagator 230
13.4 Retarded polarization propagator and linear response . . . 232
13.5 The collective excitation propagator 233
13.6 Plasmons and quasi plasmons 234
13.7 Expressing the two-particle propagator in terms of the scattering amplitude 237

14. Fermi Systems at Finite Temperature 238

14.1 Generalization of the $T=0$ case 238
14.2 Statistical mechanics in occupation number formalism . . . 239
14.3 The finite temperature propagator 241
14.4 The finite temperature vacuum amplitude 247
14.5 The pair-bubble at finite temperature 249

15. Diagram Methods in Superconductivity 254

15.1 Introduction 254
15.2 Hamiltonian for coupled electron–phonon system. 255
15.3 Short review of BCS theory 257
15.4 Breakdown of the perturbation expansion in a superconductor . 264
15.5 A brief look at Nambu formalism 267
15.6 Treatment of retardation effects by Nambu formalism . . . 271
15.7 Transition temperature of a superconductor. 272

16. Phonons From a Many-Body Viewpoint (Reprint) 275

PAGE

17. Quantum Field Theory of Phase Transitions in Fermi Systems 287
17.1 Introduction 287
17.2 Qualitative theory of phase transitions 291
17.3 Anomalous propagators and the breakdown of the perturbation series in the condensed phase 293
17.4 The generalized matrix propagator 296
17.5 Application to ferromagnetic phase in system with δ-function interaction 299
17.6 Divergence of the two-particle propagator and scattering amplitude at the transition point 302

18. Feynman Diagrams in the Kondo Problem 308
18.1 Introduction 308
18.2 Second-order (Born) approximation 311
18.3 Parquet approximation with bare propagators 316
18.4 Self-consistently renormalized *s*-electrons 317
18.5 Strong-coupling approximation with self-consistently renormalized pseudofermions and vertices 318

19. The Renormalization Group 328
19.1 Introduction 328
19.2 Review of effective interaction in the high-density electron gas . 328
19.3 Renormalization group for interaction propagators in the high-density electron gas 330
19.4 Transforming from one transformed quantity to another: the functional equation of the renormalization group . . . 331
19.5 Lie equation for the renormalization group 332
19.6 Solution of the Lie equation 333

Appendices

A. Finding fictitious particles with the canonical transformation . . 336
A. Dirac formalism 345
B. The time development operator, $U(t)$ 351
C. Finding the ground state energy from the vacuum amplitude . . 354
D. The $\tilde{U}(t)$ operator and its expansion 355
E. Expansion of the single-particle propagator and vacuum amplitude . 359
F. Evaluating matrix elements by Wick's theorem 362
G. Derivation of the graphical expansion for propagator and vacuum amplitude 367
H. The spectral density function 372
I. How the $i\delta$ factor is used 376
J. Electron propagator in normal electron–phonon system . . . 378
K. Spin wave functions 384
L. Summary of different kinds of propagators and their spectral representations and analytic properties 385
M. The decoupled equations of motion for the Green's function expressed as a partial sum of Feyman diagrams 391
N. The reduced graph expansion 395

PAGE

Answers to Exercises 402

References 414

Index 422