

Contents

PREFACE	5
1 THE GENERAL PROBLEM OF PARTICLE TRANSPORT	1
1.1 Introduction	1
1.2 Statistical notions of transport	1
1.3 Reduction to the Boltzmann equation	4
1.4 Correlations and fluctuations arising from fission	8
2 THE BOLTZMANN EQUATION FOR GAS ATOMS AND NEUTRONS	11
2.1 Introduction	11
2.2 The equation of neutron transport	11
2.3 The integral transport equation	17
2.4 Derivation of Boltzmann equation for gases	26
2.4.1 Mixtures of different gases	30
2.4.2 A canonical transformation	31
2.4.3 Other forms of the net scattering rate	32
2.5 The equilibrium solution and the H-theorem	33
2.6 An alternative derivation of the neutron transport equation	35
2.7 The continuum equations	38
2.8 The linearised Boltzmann equation for gases	42
3 BOUNDARY CONDITIONS	50
3.1 Neutron transport equation	50
3.2 Boltzmann equation for gases	52
3.2.1 Approximations to the particle-wall scattering kernel	53
3.2.2 Accommodation coefficients	56

10 CONTENTS

4 SCATTERING KERNELS	60
4.1 Neutron scattering kernels	60
4.1.1 The gas model	61
4.1.2 Scattering in crystals	65
4.2 Gas atom scattering kernels	66
4.2.1 The hard-sphere model	67
4.2.2 Maxwell molecules	70
4.3 Synthetic scattering kernels	72
4.3.1 Synthetic neutron kernels	73
4.3.2 Synthetic gas kernels	74
4.3.3 Maxwell molecules	79
4.4 Special scattering models in mixtures	83
4.5 Scattering kernels for radiation	84
5 SOME BASIC PROBLEMS IN NEUTRON TRANSPORT AND RAREFIED GAS DYNAMICS	86
5.1 Neutron transport theory	86
5.1.1 Infinite medium problem	86
5.1.2 Neutron diffusion coefficient in a non- absorbing medium	88
5.1.3 Infinite medium diffusion length problem	89
5.1.4 Finite medium diffusion length problem	90
5.1.5 The critical problem	91
5.1.6 The pulsed neutron problem	91
5.1.7 The Milne problem	92
5.1.8 Green's function and albedo problems	93
5.1.9 The Kottwitz problem	93
5.1.10 Neutron waves	94
5.1.11 Problems in non-planar geometries	95
5.2 Rarefied gas dynamics	95
5.2.1 Infinite medium problems	96
5.2.2 Temperature jump and velocity slip	101
5.2.3 Poiseuille flow	103
5.2.4 Couette flow	105
5.2.5 Heat transfer	107
5.2.6 The Rayleigh problem	109
5.2.7 Sound waves in gases	110
5.2.8 Relaxation phenomena	113
5.2.9 Free molecular flow	115
5.3 Some points of similarity	117

6 THE INTEGRAL FORM OF THE TRANSPORT EQUATION IN PLANE, SPHERICAL AND CYLINDRICAL GEOMETRIES	121
6.1 Plane geometry	121
6.1.1 Derivation of integral equation from first principles	124
6.2 Spherical geometry	126
6.2.1 Bare sphere	126
6.2.2 Two region spherical problem—the black sphere	128
6.2.3 Cross-sections that vary inversely as the radius	133
6.3 Cylindrical geometry	135
6.3.1 Uniform along the axis	135
6.3.2 Circular cylinder	137
6.3.3 Two-region cylindrical problem—the black cylinder	139
6.3.4 Cross-sections that vary inversely as the radius	143
6.4 Derivation of the integral equation by the method of characteristics	144
7 EXACT SOLUTIONS OF MODEL PROBLEMS	152
7.1 Introduction	152
7.2 Synthetic kernels	152
7.2.1 Neutron scattering	152
7.2.2 Gas atom scattering	153
7.3 Model problems in neutron transport theory	154
7.3.1 Infinite medium diffusion length problem	154
7.3.2 The Milne problem	162
7.3.3 The foil problem	188
7.3.4 One-speed critical problem: replication	195
7.3.5 General structure of solutions of the neutron transport equation	211
7.3.6 Other methods of solution	212
7.4 Model problems in rarefied gas dynamics	213
7.4.1 Velocity slip and temperature jump	213
7.4.2 Poiseuille flow	232
7.4.3 Couette flow	247
8 EIGENVALUE PROBLEMS IN TRANSPORT THEORY	265
8.1 Introduction	265
8.2 Relaxation in time	265

12 CONTENTS

8.2.1	Neutron density relaxation	267
8.2.2	Gas atom density relaxation	270
8.2.3	Extension to finite bodies	273
8.3	Relaxation in space	273
8.3.1	Neutron density relaxation	274
8.3.2	Gas atom density relaxation	276
8.3.3	Extensions to multi-dimensional transport theory	277
8.4	Wave motion	279
8.4.1	Neutron wave dispersion law	280
8.4.2	Sound wave dispersion law	282
8.4.3	Extensions to multi-dimensional transport theory	285
8.5	Analytic continuation and pseudo-discrete eigenvalues	287
8.5.1	Sound wave propagation	288
8.5.2	Diffusion length problem	292
9	COLLISION PROBABILITY METHODS	298
9.1	Introduction	298
9.2	Derivation of the basic equations	298
9.3	Single cell theory	305
9.4	Plane geometry	307
9.5	Approximate forms for collision probabilities	314
9.6	Multiple collision probability	316
9.7	Summary of collision probability techniques	319
10	VARIATIONAL METHODS	323
10.1	Introduction	323
10.2	Derivation of variational functionals	323
10.3	Eigenvalues and weighted coefficients—some examples from neutron transport theory	327
10.3.1	Critical conditions in a multiplying sphere	327
10.3.2	Escape probability	330
10.3.3	Flux depression in a plane foil	331
10.3.4	Milne problem extrapolated endpoint	335
10.3.5	Extrapolation distance at the surface of a black sphere	340
10.4	Applications of the method to rarefied gas dynamics	343
10.4.1	Heat transfer	343
10.4.2	Poiseuille flow	349
10.4.3	Temperature jump and velocity slip	351

10.4.4 Viscosity and thermal conductivity	357
10.5 The method of overlapping spectra	362
10.5.1 Reactor cell calculations	363
10.5.2 Shock-wave structure	368
11 POLYNOMIAL APPROXIMATIONS	376
11.1 Introduction	376
11.2 Full-range expansions in neutron transport theory	380
11.2.1 The P_N approximation	380
11.2.2 Kinetic theory and the moment equations	392
11.3 Half-range expansions	399
11.3.1 Neutron transport theory—Double- P_l method	400
11.3.2 Kinetic theory	421
11.3.3 Method of discrete ordinates and general comments	421
INDEX	427