## **GENERALIZED HARMONIC ANALYSIS<sup>1</sup>**

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Bibliography

## INTRODUCTION.

Numerous important branches of mathematics and physics concern themselves with the asymptotic behavior of functions for very large or very

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