

Contents

1. General Preliminaries	1
1.1 Overview	1
1.1.1 Subjects of Statistical Mechanics	1
1.1.2 Approach to Equilibrium	3
1.2 Averages	4
1.2.1 Probability Distribution	4
1.2.2 Averages and Thermodynamic Fluctuation	10
1.2.3 Averages of a Mechanical System – Virial Theorem	12
1.3 The Liouville Theorem	17
1.3.1 Density Matrix	17
1.3.2 Classical Liouville’s Theorem	19
1.3.3 Wigner’s Distribution Function	22
1.3.4 The Correspondence Between Classical and Quantum Mechanics	25
2. Outlines of Statistical Mechanics	28
2.1 The Principles of Statistical Mechanics	28
2.1.1 The Principle of Equal Probability	28
2.1.2 Microcanonical Ensemble	29
2.1.3 Boltzmann’s Principle	30
2.1.4 The Number of Microscopic States, Thermodynamic Limit	32
a) A Free Particle	32
b) A Perfect Gas	33
c) Spin System	35
d) The Thermodynamic Limit	37
2.2 Temperature	37
2.2.1 Temperature Equilibrium	37
2.2.2 Temperature	39
2.3 External Forces	41
2.3.1 Pressure Equilibrium	41
2.3.2 Adiabatic Theorem	43
a) Adiabatic Change	43
b) Adiabatic Theorem in Statistical Mechanics	45
c) Adiabatic Theorem in Classical Mechanics	47

2.3.3	Thermodynamic Relations	50
2.4	Subsystems with a Given Temperature	51
2.4.1	Canonical Ensemble	51
2.4.2	Boltzmann-Planck's Method	54
2.4.3	Sum Over States	57
2.4.4	Density Matrix and the Bloch Equation	58
2.5	Subsystems with a Given Pressure	61
2.6	Subsystems with a Given Chemical Potential	63
2.6.1	Chemical Potential	63
2.6.2	Grand Partition Function	65
2.7	Fluctuation and Correlation	66
2.8	The Third Law of Thermodynamics, Nernst's Theorem	68
2.8.1	Method of Lowering the Temperature	70
3.	Applications	72
3.1	Quantum Statistics	72
3.1.1	Many-Particle System	72
3.1.2	Oscillator Systems (Photons and Phonons)	74
3.1.3	Bose Distribution and Fermi Distribution	78
a)	Difference in the Degeneracy of Systems	79
b)	A Special Case	80
3.1.4	Detailed Balancing and the Equilibrium Distribution	82
3.1.5	Entropy and Fluctuations	84
3.2	Perfect Gases	88
3.2.1	Level Density of a Free Particle	88
3.2.2	Perfect Gas	90
a)	Adiabatic Change	91
b)	High Temperature Expansion	91
c)	Density Fluctuation	92
3.2.3	Bose Gas	93
3.2.4	Fermi Gas	95
3.2.5	Relativistic Gas	98
a)	Photon Gas	99
b)	Fermi Gas	99
c)	Classical Gas	100
3.3	Classical Systems	100
3.3.1	Quantum Effects and Classical Statistics	100
a)	Classical Statistics	102
b)	Law of Equipartition of Energy	103
3.3.2	Pressure	104
3.3.3	Surface Tension	106
3.3.4	Imperfect Gas	107
3.3.5	Electron Gas	112

3.3.6 Electrolytes	114
4. Phase Transitions	118
4.1 Models	118
4.1.1 Models for Ferromagnetism	118
4.1.2 Lattice Gases	120
4.1.3 Correspondence Between the Lattice Gas and the Ising Magnet	121
4.1.4 Symmetric Properties in Lattice Gases	124
4.2 Analyticity of the Partition Function and Thermodynamic Limit	126
4.2.1 Thermodynamic Limit	126
4.2.2 Cluster Expansion	129
4.2.3 Zeros of the Grand Partition Function	129
4.3 One-Dimensional Systems	132
4.3.1 A System with Nearest-Neighbor Interaction	132
4.3.2 Lattice Gases	133
4.3.3 Long-Range Interactions	135
4.3.4 Other Models	137
4.4 Ising Systems	137
4.4.1 Nearest-Neighbor Interaction	137
a) One-Dimensional Systems	138
b) Many-Dimensional Systems	138
c) Two-Dimensional Systems	140
d) Curie Point	141
4.4.2 Method of Matrix	142
a) One-Dimensional Ising System	142
b) Two-Dimensional Ising Systems	144
4.4.3 Zeros on the Temperature Plane	149
4.4.4 Spherical Model	150
4.4.5 Eight-Vertex Model	151
4.5 Approximate Theories	153
4.5.1 Molecular Field Approximation	153
4.5.2 Bethe Approximation	155
4.5.3 Low and High Temperature Expansions	158
4.6 Critical Phenomena	160
4.6.1 Critical Exponents	160
4.6.2 Phenomenological Theory	164
4.6.3 Scaling	168
4.7 Renormalization Group Method	171
4.7.1 Renormalization Group	171
4.7.2 Fixed Point	174

5. Ergodic Problems	177
5.1 Some Results from Classical Mechanics	178
5.1.1 The Liouville Theorem	178
5.1.2 The Canonical Transformation	178
5.1.3 Action and Angle Variables	178
5.1.4 Integrable Systems	181
5.1.5 Geodesics	183
5.2 Ergodic Theorems	185
5.2.1 Birkhoff's Theorem	188
5.2.2 Mean Ergodic Theorem	189
5.2.3 Hopf's Theorem	191
5.2.4 Metrical Transitivity	191
5.2.5 Mixing	192
5.2.6 Khinchin's Theorem	193
5.3 Abstract Dynamical Systems	195
5.3.1 Bernoulli Schemes and Baker's Transformation	195
5.3.2 Ergodicity on the Torus	198
5.3.3 K-Systems (Kolmogorov Transformation)	199
5.3.4 C-Systems	202
5.4 The Poincaré and Fermi Theorems	204
5.4.1 Bruns' Theorem	204
5.4.2 Poincaré-Fermi's Theorem	204
5.5 Fermi-Pasta-Ulam's Problem	206
5.5.1 Nonlinear Lattice Vibration	206
5.5.2 Resonance Conditions	209
5.5.3 Induction Phenomenon	212
5.6 Third Integrals	216
5.7 The Kolmogorov, Arnol'd and Moser Theorem	222
5.8 Quantum Mechanical Systems	227
5.8.1 Theorems in Quantum Mechanical Systems	227
5.8.2 Chaotic Behavior in Quantum Systems	233
5.8.3 Adiabatic Processes and Susceptibility	235
General Bibliography	239
References	241
Subject Index	245