

## CONTENTS

### PART A. INTRODUCTION: SYNERGETICS AND NON-EQUILIBRIUM PHASE TRANSITIONS

#### Synergetics

H. Haken

1. Introduction 1
2. Outline of the general approach 3
3. A brief outline of the mathematical approach 4
4. Some simple examples 8
5. Generalized Ginzburg-Landau equations 10
6. Some further applications 16
7. Hierarchies of temporal patterns: from oscillations to chaos 17
8. Outlook 21

#### Phase transition analogies: magnets, lasers and fluid flows

J. Gea-Banacloche, M.O. Scully and M.G. Velarde

1. Introduction 25
2. Laser threshold- Second-order phase transition analogy 26
3. Laser with saturable absorber - First-order phase transition 32
4. From lasers to fluid flows 33

### PART B. LASERS AND QUANTUM OPTICS

#### Collective phenomena in Quantum Optics

F.T. Arecchi

- Introduction 37
1. Physics of stimulated emission processes 39
2. What is quantum and what is classical in laser physics. 48
3. Bifurcations in non-equilibrium systems 57
4. Photon statistics and the laser threshold 60
5. Turbulence in quantum optics. 66

Optical Bistability and related topics

L.A. Lugiato and L. M. Narducci

- 1. Introduction 73
- 2. A practical optical bistable device: Recent advances 76
- 3. Theory of optical bistability in a ring cavity 77
  - 3.1 Steady-state behavior in absorptive optical bistability with zero cavity detuning 81
  - 3.2 The effect of dispersion - Kerr medium 82
- 4. The mean field model of optical bistability 87
- 5. Self-pulsing and chaotic behavior 89
- 6. Instability modes 91
  - 6.1 Resonant-mode instability 91
  - 6.2 Off-resonance mode instability 92

Laser with intracavity absorber: Q-switching and multistable nonlinear oscillations and chaos

M.G. Velarde and J.C. Antoranz

- 1. Introduction 103
- 2. Phase diagram 104
- 3. Soft oscillatory lasing 105
- 4. Q-switching and coexistence between oscillations and cw lasing 106
- 5. Coexistence of various oscillations including Q-switching 108
- 6. A Lorenz-like strange (aperiodic) solution and its fractal dimension 111
- 7. A Feigenbaum period-doubling cascade, the subsequent strange (aperiodic) attractor and its fractal dimension 113

Non-linear operation of  $\text{CO}_2$  lasers with intracavity saturable absorbers

E. Arimondo

- Introduction 117
- Experimental apparatus 117
- Experimental results 119
- Theoretical model 124
- Conclusion 134

Structurally stable bifurcations in optical bistability

D. Armbruster and G. Dangelmayr

- 1. Introduction 137
- 2. Self-pulsing and optical bistability 138
- 3. Polarization switching 141

PART C. ACTION OF INTENSE LASER FIELDS

Allowed nuclear beta decay in an intense laser field

R.R. Schlicher, W. Becker and M.O. Scully

- Introduction 145

Wavefunctions	148
Transition rate	156
Alternative derivations of the quasiclassical Transition rate	160
Electron distributions	163
Nuclear lifetime	169
The nonclassical regime	172
Summary	176

## PART D. INSTABILITIES AND CONVECTION IN LIQUIDS AND LIQUID CRYSTALS

### Thermohydrodynamic instabilities : Buoyancy-thermocapillary convection

M.G. Velarde and J.L. Castillo	
1. Introduction	179
2. Interfacial convection: heuristic arguments	183
3. Buoyancy - thermocapillary equations	185
4. Necessary conditions for the onset of convection	188
5. Sufficient conditions for the onset of convection	190
6. Comments	194

### Thermohydrodynamic instability in nematic liquid crystals: a summary of arguments and conditions for some simple geometries

I. Zuñiga and M.G. Velarde	
1. Introduction	197
2. Rayleigh-Benard convection	197
3. Rotating annulus convection with radial temperature gradients	203

### Neutron scattering studies of phase transitions in equilibrium and nonequilibrium systems

T. Riste	
1. Introduction	207
1a. Neutron scattering	207
1b. Phase transition in equilibrium	208
2. Neutron scattering studies of nonequilibrium phase transitions	213

### Electrothermal instabilities in dielectric liquids

A. Castellanos, P. Atten and M.G. Velarde	
1. Introduction	223
2. Electrothermal equations	224
3. The onset of steady convection in a simple approach	225
4. Convective flow and the phase transition picture	227
5. Overstability	231

### Electrohydrodynamic instabilities in nematic with homeotropic boundaries

J. Grupp	
Introduction	235

EHD convention flow in samples with homeotropic boundaries	235
Experiments and results	237

PART E. CONVECTION DIFFUSION AND REACTION

Gyrotactic buoyant convection and spontaneous pattern formation in algal cell cultures

J.O. Kessler	
Introduction	241
Gyrotaxis and focusing	242
Convection pattern formation	243
Cooperative effects in the time domain	245
Other organisms and conditions	245
Discussion	245
Applications	246

Experimental investigations of precipitation patterns

S.C. Muller	
1. Introduction	249
2. Spatial and temporal sequence of events in a Liesegang system	251
3. Pattern formation in the presence of low concentration gradients	254
4. Pattern formation in initially uniform colloids	255
5. Complex patterns in periodic precipitation processes	256
6. Conclusion	256

PART F. COMBUSTION

Optical diagnosis in flows - Applications - Experiments in Combustion

L. Boyer	
Introduction	259
A powerful non invasive tool: the interaction of light with matter	262
Some applications of Rayleigh and Mie scatterings in flow studies	271
Experimental study of the flow-flame interaction	283

Theory of gaseous combustion

P. Clavin and A. Liñan	
I. Basic considerations	291
I.1 Two feed-back mechanisms	291
I.2 The conservation equations	292
I.3 The adiabatic temperature of combustion	293
I.4 The two different kinds of combustion processes	294
I.5 The large activation energy	296

II. Premixed flames	297
II.1 Position of the problem	297
II.2 Existence and unicity	299
II.3 The asymptotic expansion	310
II.4 Dynamics of flame front	314
III. Diffusion flames	319
III.1 Position of the problems	319
III.2 Ignition regime	324
III.3 Diffusion controlled regime	326
III.4 Extinction regime	329
Bifurcation in Heterogeneous Combustion	
L. de Luca, G. Riva and C. Bruno	
1. Introduction	339
2. Mathematical model	339
3. Bifurcation diagrams	341
4. Numerical solution	345
5. Conclusions	346

## PART G. INSTABILITIES AND NONLINEAR PHENOMENA IN SOLIDS

### Thermoelasticity and mechanical instabilities

C.E. Bottani and G. Caglioti	
Introduction and Summary	349
I. The thermoelastic effect	350
II. The thermoelastic-plastic transition in metals	352
II.1 Introduction	352
II.2 The yield point as the critical point of a dynamical instability	352
II.3 "Thermal emission" in 100 Cr6 steel	356
II.4 Conclusions	360

### Non-Equilibrium effects seen in molecular dynamics calculations of Shock waves in Solids

F.E. Walker, A.M. Karo and J.R. Hardy	
Introduction	363
Discussion	364
Conclusions	369

## PART H. DETERMINISTIC (CONTINUOUS AND DISCRETE) MATHEMATICS OF NONLINEAR PROBLEMS

### Current topics in reaction-diffusion systems

P.C. Fife	
1. Introduction	371
2. The simplest wave fronts	373
3. Slowly varying fronts	375
4. Coupling with another reactant; propagator-controller systems	377
5. Accounting for dissipation of V	385

6. Target patterns for the Belousov-Zhabotinsky reagent	386
7. The generation of spirals	390
8. Compound layers and stationary solutions	393
9. Small wave trains and associated solutions	397
10. Piecing together a global picture for a model problem	407
Discrete nonlinear dynamics	
S. Grossmann	
1. Introduction	413
2. Why discrete dynamics?	413
3. Experiments	415
4. Trifurcation?	418
5. Density of states, renormalization, and chaos map	419
6. Random numbers?	422
7. Period-doubling renormalization group	424
8. Deterministic Brownian motion?	427
9. Windows of nondiffusive states	430
10. Anomalous diffusion?	431
11. Ornstein-Uhlenbeck process	432
Deterministic diffusion - A quality of Chaos	
T. Geisel	
1. Introduction	437
2. The onset of diffusion	439
2.1 An example	439
2.2 Master equation	441
2.3 Critical properties of the diffusion coefficient	445
3. Excess noise for intermittent diffusion	447
4. Anomalous diffusion	450
5. Diffusion in two dimensions	451
 <u>PART I. STOCHASTIC DESCRIPTION OF NON LINEAR PROBLEMS</u>	
Stochastic space-time problems	
L. Arnold and P. Kotelenetz	
1. Reaction-diffusion models	455
1.1 Introduction and motivation	455
1.2 Deterministic model. Sobolev spaces	456
1.3 Markov jump processes	457
1.4 Stochastic model of reaction with diffusion	459
1.5 Consistency of the stochastic and deterministic model (Thermodynamic limit)	460
1.6 Consistency of the stochastic and deterministic model (Continued)	461
2. Stochastic partial differential equations	463
2.1 Motivation	463
2.2 Deterministic evolution equations. Semigroups	464
2.3 Wiener process and stochastic integrals in Hilbert space	466
2.4 Stochastic evolution equations	468

3. SPDE in reaction-diffusion problems	468
3.1 Introduction. Van Kampen's approximation	468
3.2 Central limit theorem	469
Stochastic theory of transition phenomena in nonequilibrium systems	
G. Nicolis and C. Van den Broeck	
1. Introduction	473
1A. General formulation	473
1B. Simple models	476
1C. The importance of fluctuations	478
2. Stochastic formulation	479
2A. The master equation	479
2B. Birth and death processes	479
2C. Spatially distributed systems	481
2D. Some important limits	482
2E. Stochastic thermodynamics	484
3. Primary bifurcation	487
3A. Critical behavior	487
3B. Nucleation	491
4. The onset of spatial correlations	496
4A. Reaction-Diffusion systems	496
4B. Heat conduction in nonequilibrium	499
5. Transient phenomena	500
Non-equilibrium systems with random control parameters	
M. San Miguel and J.M. Sancho	
Introduction	505
External noise	506
Mathematical description	506
Non-white noise triggered oscillations in nonlinear chemical process	
F.J. de la Rubia, J.J. García-Sanz and M.G. Velarde	
1. Introduction	513
2. Deterministic model	513
3. Stochastic model	514
4. Numerical results	517
5. Discussion and conclusions	520
About some simple Fokker-Planck models	
J.J. Brey	
Introduction	523
A very simple model	523
Expansion in the nonlinearity parameter	526
Some comments	527

Dynamics of symmetry breaking: phase coherence in finite and random system	
F. de Pasquale	
Introduction	529
1. Spherical limit of TDGL	531
2. Size and dimensionality effects	536
3. Dynamics of the random field instability	541
4. Conclusion	543
Pictures of participants	545
Participants	547
Author Index	559
Subject Index	567