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## Values of the Physical Constants

[From Symbols, Unit and Nomenclature in Physics, Document U.I.P.11  
(S.U.N. 65 — 3) of the IUPAP (1965)]

Velocity of light in vacuo

$$c = (2.997925 \pm 0.000003) \cdot 10^{10} \text{ cm sec}^{-1}$$

Avogadro's number (Loschmidt's number)

$$N_L = (3.02252 \pm 0.00028) \cdot 10^{23} \text{ mol}^{-1}$$

Electronic charge

$$|e| = (4.80298 \pm 0.00020) \cdot 10^{-10} \text{ esu}$$

Rest mass of the electron

$$m_e = (9.1091 \pm 0.0004) \cdot 10^{-28} \text{ g}$$

Rest mass of the proton

$$m_p = (1.67252 \pm 0.00008) \cdot 10^{-24} \text{ g}$$

Rest mass of the neutron

$$m_n = (1.67482 \pm 0.00008) \cdot 10^{-24} \text{ g}$$

Planck's constant

$$\hbar = (6.6256 \pm 0.0005) \cdot 10^{-2} \text{ erg sec}$$

Boltzmann's constant

$$k = (1.38054 \pm 0.00018) \cdot 10^{-16} \text{ erg deg}^{-1}$$