

Contents

<i>Preface</i>	<i>xv</i>
<i>I. Entropy Optimization Principles</i>	<i>1</i>
1.1 The Rationale for Entropy Optimization Principles	1
1.1.1 Some Basic Questions about Entropy Optimization Principles	1
1.1.2 Why Do We Talk of Entropy Maximization Principles?	6
1.1.3 A Seeming Paradox	7
1.1.4 Uncertainty or Entropy?	7
1.2 Another Entropy Optimization Principle	10
1.3 A Diversity of Optimization Principles	14
1.4 A Diversity of Applications	17
1.5 Overview of the Book	20
<i>II. Jaynes' Maximum Entropy Principle</i>	<i>23</i>
2.1 Introduction	23
2.2 Properties of Shannon's Measure of Entropy	27
2.2.1 Exercises	35

2.3	Jaynes' Maximum Entropy Principle: MaxEnt	36
2.3.1	Exercises	41
2.4	Jaynes' Maximum Entropy Formalism (MaxEnt)	42
2.4.1	The Maximum Entropy Probability Distribution (MEPD)	42
2.4.2	Convexity of λ_0 as a function of $\lambda_1, \lambda_2, \dots, \lambda_m$	44
2.4.3	Value of Maximum Entropy	46
2.4.4	Concavity of S_{\max} as a Function of a_1, a_2, \dots, a_m	46
2.4.5	Interpretation of Lagrange Multipliers	48
2.4.6	Alternative Proof That MaxEnt Gives Globally Maximum Values of Entropy	48
2.4.7	Jaynes' Entropy Concentration Theorem	49
2.4.8	Non-Negativity of Maximizing Probabilities	52
2.4.9	Inequality Moment Constraints	53
2.4.10	Exercises	54
2.5	MaxEnt for Continuous-Variate Distributions	55
2.5.1	Need for Measure of Entropy for Continuous-Variate Probability Distributions	55
2.5.2	Measure of Entropy for the Continuous-Variate Probability Distribution	56
2.5.3	Jaynes' Maximum Entropy Formalism for the Continuous-Variate Case	59
2.5.4	Extension of the Range of Integration	62
2.5.5	Exercises	64
2.6	Characterization of Continuous-Variate Distributions	65
2.6.1	Maximum Entropy Probability Distribution over the Range $[a, b]$	65
2.6.2	Maximum Entropy Probability Distributions over the Range $[0, \infty)$	66
2.6.3	Maximum Entropy Probability Distributions over the Range $(-\infty, \infty)$	67
2.6.4	Multivariate Normal Distribution as Maximum Entropy Distribution	68
2.6.5	Summary of Results	69
2.6.6	Exercises	70
2.7	The Extended MaxEnt	71
2.7.1	Exercises	75

III. Applications of Jaynes' Maximum Entropy Principle	77
3.1 Introduction	77
3.2 Statistical Mechanics Distributions	78
3.2.1 The Maxwell–Boltzmann (MB) Distribution	79
3.2.2 The Bose–Einstein (BE) Distribution	85
3.2.3 The Fermi–Dirac (FD) Distribution	88
3.2.4 The Intermediate Statistics (IS) Distribution	90
3.2.5 Some Applications of the Intermediate Statistics Distribution	92
3.2.6 Some Remarks on Distributions of Statistical Mechanics	93
3.2.7 Exercises	96
3.3 MaxEnt and the Laws of Thermodynamics	98
3.3.1 Exercises	107
3.4 MaxEnt and Economics	108
3.4.1 Variation of Population Density and Rents in a City	108
3.4.2 Population Distribution Subject to Travel Budget Constraint	109
3.4.3 The Maximization of Entropy and the Minimization of Travel Cost	114
3.4.4 Maximization of Entropy and Minimization of the Expected Rent and the Travel Budget	116
3.4.5 Primary and Secondary Entropies	119
3.4.6 Exercises	121
3.5 Applications to Regional and Urban Planning	123
3.5.1 Introduction	123
3.5.2 A Transportation Problem	124
3.5.3 Estimating the Number of Visitors to a Shopping Plaza	128
3.5.4 Estimating the Number of Shoppers Living in Different Colonies	129
3.5.5 More General Transportation Problems	130
3.5.6 International Trade Model	131
3.5.7 Exercises	132
3.6 Further Applications to Statistics	133
3.6.1 Introduction	133

3.6.2	Characterization of Probability Distributions as MaxEnt Distributions	134
3.6.3	Method of Maximum Likelihood and Estimation of a_i 's	136
3.6.4	Significance of the Previous Result: Equivalence of Gauss's and the Likelihood Principles of Estimation	137
3.6.5	Principle of Maximum Likelihood from MaxEnt	139
3.6.6	Comparison of Fisher's and MaxEnt Methods of Estimation	139
3.6.7	Comparison with Pearson's Method of Moments	140
3.6.8	Information in Contingency Tables	142
3.6.9	Log Linear Models	148
3.6.10	Exercises	150
IV.	<i>Kullback's Minimum Cross-Entropy Principle</i>	151
4.1	Introduction	151
4.2	Some Properties of Kullback–Leibler's Measure of Cross-Entropy	155
4.2.1	Exercises	161
4.3	Kullback's Minimum Cross-Entropy Principle (MinxEnt)	163
4.3.1	The Relationship of Kullback's MinxEnt to Jaynes' MaxEnt	163
4.3.2	Formalism of Kullback's MinxEnt	166
4.3.3	Exercises	171
4.4	Some Discrete-Variate MinxEnt Distributions	171
4.4.1	When a Variate Takes Only a Finite Set of Values and the Mean Is Prescribed	171
4.4.2	When the Variate Takes a Countably Infinite Set of Values and the Mean Alone Is Prescribed	174
4.4.3	Discrete Multivariate MinxEnt Probability Distributions	178
4.4.4	Discussion of the Characterization of Discrete Variate Distributions	181
4.4.5	Exercises	186
4.5	Some Further Applications of the MinxEnt Principle	188

4.5.1	Estimating Populations of Residential Colonies	188
4.5.2	Transportation Problem Revisited	189
4.5.3	International Trade	191
4.5.4	Minimizing Risk in Portfolio Analysis	191
4.5.5	Measurement of Dependence among Random Variates	197
4.5.6	Exercises	201
V.	<i>Further Applications of MaxEnt and MinxEnt Principles</i>	205
5.1	Introduction	205
5.1.1	The Problem of Pattern Recognition	207
5.1.2	Choosing A So As to Minimize the Loss of Information	210
5.1.3	Choosing A So As to Minimize the Loss of Power of Discrimination	212
5.1.4	Choosing A So As to Minimize the Dependence among y_1, y_2, \dots, y_m	215
5.1.5	Choosing A So As to Maximize the Distinguishability of y_1, y_2, \dots, y_m	217
5.1.6	Pattern Recognition As a Quest for Minimum Entropy	218
5.1.7	Exercises	222
5.2	Applications to Non-Linear Spectral Analysis	222
5.2.1	Introduction to Time Series	222
5.2.2	The Maximum Entropy Approach	227
5.2.3	Exercises	246
5.3	Application of MaxEnt to Queuing Theory	247
5.3.1	The Problem	247
5.3.2	The Arrival and Service Time Distributions	248
5.3.3	Some Results from Queuing Theory	249
5.3.4	Some MaxEnt System Size Distributions	252
5.3.5	The Inverse Problem	255
5.3.6	A New Approach to Queuing Theory	256
5.3.7	Another Application of MaxEnt to Queuing Theory	256
5.3.8	Approximating a Given Probability Distribution by MaxEnt Distributions	259
5.3.9	Exercises	266

5.4	Applications of Entropy Optimization Principles to Parameter Estimation	266
5.4.1	The Problem	266
5.4.2	Solution When the Only Information Available Is About the Form of the Density Function	267
5.4.3	First Method When There Is Information in the Form of a Random Sample: Derivation of Principle of Maximum Likelihood	269
5.4.4	Second Method When There Is Information in the Form of a Random Sample: Principle of Maximum Equality	270
5.4.5	Third Method When There Is Information in the Form of a Random Sample: Use of Principle of Least Information	271
5.4.6	Comparison of Second and Third Methods	274
5.4.7	Comparison with the First Method	275
5.4.8	Fourth Method When Information Is Available in the Form of a Random Sample	275
5.4.9	Fifth and Sixth Methods When Information Is Available in the Form of a Random Sample	277
5.4.10	Estimation of θ When Proportions in Different Specified Intervals Are Given	279
5.4.11	Exercises	281
VI.	<i>New Entropy Optimization Principles</i>	283
6.1	New Entropy Optimization Principles	283
6.1.1	Possible Motivations for New Entropy Optimization Principles	283
6.2	Entropy Optimization Principles	285
6.2.1	Jaynes' Maximum Entropy Principle (MaxEnt Principle)	286
6.2.2	Generalization of Jaynes' Maximum Entropy Principle (GMEP or GEN MaxEnt Principle)	286
6.2.3	Kullback's Minimum Cross-Entropy Principle (MinxEnt Principle)	287

6.2.4	Generalization of Kullback's Minimum Cross-Entropy Principle (GEN MinxEnt Principle)	287
6.2.5	First Inverse Maximum Entropy Principle	287
6.2.6	Second Inverse Maximum Entropy Principle	287
6.2.7	Third Inverse Maximum Entropy Principle	288
6.2.8	First Inverse Minimum Cross-Entropy Principle	288
6.2.9	Second Inverse Minimum Cross-Entropy Principle	288
6.2.10	Third Inverse Minimum Cross-Entropy Principle	288
6.2.11	First Minimum Interdependence Principle	288
6.2.12	Second Minimum Interdependence Principle	288
6.2.13	Minimax Entropy Principle	289
6.2.14	Maximin Cross-Entropy Principle	289
6.2.15	The Minimum Loss of Information Principle	289
6.2.16	The Minimum Loss of Power of Discrimination Principle	290
6.2.17	The Minimum Entropy Principle	290
6.2.18	The Maximum Cross-Entropy Principle	290
6.2.19	The Extended Maximum Entropy Principle	290
6.2.20	The Extended Minimum Cross-Entropy Principle	291
6.2.21	The Second Extended Maximum Entropy Principle	291
6.2.22	The Second Extended Minimum Cross-Entropy Principle	291
6.2.23	Discussion of the New Principles	292
6.3	The Entropy Optimization Postulate	297
6.3.1	The Optimization Postulate	297
6.3.2	The Generalized Entropy Optimization Principle	298
6.4	Reasons for Considering Generalized Measures of Entropy and Cross-Entropy	298
6.5	Reasons for Considering Inverse Principles	301

6.6	Reasons for Considering Dual Problems	302
6.7	The Principle of Minimum Interdependence	303
VII. Generalized Principles of Maximum Entropy and Minimum Cross-Entropy		307
7.1	Introduction	307
7.2	Some Measures of Directed Divergence	311
7.2.1	Measures of Directed Divergence	311
7.2.2	Csiszer's Family of Measures of Directed Divergence	312
7.2.3	Special Cases of Csiszer's Measure	314
7.2.4	Two Related Measures of Directed Divergence	315
7.2.5	A Generalization of Csiszer's Measure of Directed Divergence	316
7.2.6	Non-Negativity of Minimizing Probabilities	317
7.2.7	Exercises	319
7.3	Some Generalized Measures of Entropy	321
7.3.1	Exercises	324
7.4	Examples of Generalized Optimization Principles	325
7.4.1	Some Conclusions from These Examples	336
7.4.2	Exercises	337
7.5	Some Applications of Generalized Measures	339
7.5.1	Introduction	339
7.5.2	Use of Generalized Entropies in Marketing	341
7.5.3	Measurement of Risk in Portfolio Analysis	344
7.5.4	The Generalized Logit Models	345
7.5.5	Exercises	346
7.6	Historical Development of Generalized Measures	347
VIII. The Four Inverse Maximum Entropy Principles		353
8.1	Introduction	353
8.2	The First Inverse Maximum Entropy Principle	355
8.2.1	Existence of Solutions	355
8.2.2	Uniqueness of the Solution	358

8.2.3	MaxEnt Characterizing Moments for Some of the Well-Known Distributions	359
8.2.4	Exercises	360
8.3	The Second Inverse Maximum Entropy Principle	361
8.3.1	Motives for Generating Generalized Measures of Entropy	361
8.3.2	A Generalized Entropy Measure from Consideration of Population Dynamics	362
8.3.3	Special Cases of Generalized Measures of Entropy	364
8.3.4	Generating Appropriate Generalized Entropy Measures for Innovation Diffusion Models	368
8.3.5	Generating Appropriate Generalized Entropy Measures from Mathematical Models in Other Fields	370
8.3.6	Some Remarks on Generalized Measures of Entropy and Mathematical Models	371
8.3.7	Exercises	372
8.4	Second Inverse Principle: Generating Useful Measures of Entropy	375
8.4.1	Introduction	375
8.4.2	A Family of Probability Distributions	375
8.4.3	A General Class of Families of Probability Distribution	376
8.4.4	Special Cases	376
8.4.5	The Existence of Solutions to Problems for the Second Inverse Maximum Entropy Principle	379
8.4.6	Uniqueness of Solution for the Second Inverse Maximum Entropy Principle	380
8.4.7	Exercises	381
8.5	The Third and Fourth Inverse Maximum Entropy Principles	382
8.5.1	The First Example	382
8.5.2	The Second Example	385
8.5.3	Exercises	387
8.6	Generating Generalized Measures	388
8.6.1	Generation Of Csiszer's Measure of Cross-Entropy	389
8.6.2	Measure of Entropy When There Are Inequality Constraints on Probabilities	391

8.6.3	A New Derivation of Fermi–Dirac and Intermediate Statistics Distributions of Statistical Mechanics	393
8.6.4	Exercises	395
8.7	Inverse MaxEnt and MinxEnt Principles in Spectral Analysis	396
8.7.1	The Inverse MaxEnt Principle	396
8.7.2	Inverse Minimum Cross-Entropy Principle	398
	<i>Bibliography</i>	<i>401</i>
	<i>Index</i>	<i>405</i>