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*Note regarding references in the text.*

Chapters are divided into sections 1, 2, . . . , sections are divided into subsections a, b, . . . , and subsections may be divided into sub-subsections i, ii, . . . . The first equation of any section bears number (1), and is not preceded by a number indicating the section or chapter in question. If, however, somewhere in the text is referred to an equation of a different section, this number precedes the number of this equation. References to equations of a different chapter are preceded, moreover, by a Roman numeral corresponding to the chapter concerned. Numbers of formulae are separated by a comma. Where a series of numbers occur they are elided. Thus one has, for instance, (1, 3–5), (1.3–5), (X.1.1–3, 5) referring respectively to formulae (1), (3), (4) and (5) of the same section, to formulae (3), (4) and (5) of section 1 and to formulae (1), (2), (3) and (5) of section 1 of chapter X.