

# Contents

Contributors	v
General Preface	vii
Preface to Volume 6	ix
<b>1. The Renormalization Group—Introduction</b>	
KENNETH G. WILSON	
References	1
<b>2. The Critical State, General Aspects</b>	
F. J. WEGNER	
I. Introduction	8
II. The Renormalization Group	11
A. Order parameter, critical exponents	11
B. From discrete to continuous models	14
C. Scale invariance and basic properties of the renormalization group	17
D. Definitions and notations	18
E. Renormalization group equation with smooth momentum cut-off	22
F. Other renormalization group transformations	27
III. Linearized Theory	29
A. Fixed point, linearized renormalization group equations	29
B. Redundant operators	34
C. Scaling of the free energy	41
D. Correlation functions in momentum space	45
E. Correlation functions in coordinate space	49
F. Trivial (Gaussian) fixed point	54
G. Comments	60
IV. Nonlinear Theory: The Nontrivial Fixed Point	65
A. The nonlinear term	65
B. The nontrivial fixed point in order $\varepsilon$	70
C. Exponent $\eta$	73
D. Isotropic $n$ -component model	77
V. Nonlinear Theory: Homogeneous Systems	80
A. Scaling fields	80
B. Invariance properties	85
C. Universality	88
D. Coexistence curve	91
E. Logarithmic anomalies	94
F. The limit case $y_E = 0$ : Phase transitions of infinite order	98

<b>VI. Nonlinear Theory: Correlations</b>	<b>105</b>
A. Order parameter correlations	105
B. Recursion equation for correlation functions	107
C. Correlation functions for finite wave-lengths near $T_c$	111
D. Scaling fields for inhomogeneous perturbations, universality of scaling functions	115
References	122
<b>3. Field Theoretical Approach to Critical Phenomena</b>	
E. BREZIN, J. C. LE GUILLOU AND J. ZINN-JUSTIN	
<b>I. Introduction</b>	<b>127</b>
A. Characterization of the static critical behaviour	129
B. Correlation and thermodynamic scaling	131
<b>II. Perturbation Theory</b>	<b>132</b>
A. Perturbation expansion of the partition function	133
B. Generating functionals for the connected and the one-particle irreducible correlation functions	134
C. Loop expansion	136
D. Dimensional continuation of Feynman diagrams	139
<b>III. Renormalization Scheme</b>	<b>141</b>
A. The problem of divergences in local field theories	141
B. Regularization	143
C. Renormalization	143
D. Composite operators	148
E. Equations of motion	153
F. Massless theory	156
<b>IV. Mean Field Theory and the Role of Dimension Four</b>	<b>157</b>
A. Mean field theory	157
B. Beyond mean field theory	165
C. Validity of Landau theory above four dimensions Four	168
D. Remark concerning tricritical points	169
E. Perturbation theory for an arbitrary spin Hamiltonian	169
<b>V Critical Phenomena near Dimension Four</b>	<b>170</b>
A. Renormalization group equations at $T = T_c$	171
B. Consequences of the renormalization group equations	174
<b>VI. Renormalized Theory: Derivation of Scaling Laws Above the Critical Temperature</b>	<b>177</b>
A. Renormalization at zero momentum	177
B. Renormalization group equations for the critical theory	181
C. Expansion around the critical theory	183
D. Scaling laws above $T_c$	184
E. Correlation functions involving $S^2$ insertions	186
<b>VII. Scaling Laws below the Critical Temperature</b>	<b>188</b>
A. Equation of state	188
B. Correlation functions below $T_c$ for Ising-like systems	191
C. Longitudinal and transverse susceptibilities below $T_c$ for an $n$ -component system	192

VIII. Corrections to Scaling Laws	193
A. Deviations from scaling in the $S^4$ -theory below four dimensions	193
B. Logarithmic corrections in four dimensions	195
C. Deviations from scaling induced by irrelevant even operators	198
D. Renormalization group equations for the tricritical behaviour	200
IX. The $\epsilon$ -Expansion : Calculations and Results	202
A. Calculation of the critical exponents	203
B. The equation of state	208
C. The critical spin-spin correlation function for arbitrary magnetization and temperature	212
D. Universal ratios derived from the equation of state	213
E. Other universal ratios of amplitudes in Ising-like systems	215
X. Renormalization Group for the $n$ -Vector Model in the Large $n$ -Limit	217
A. Algebraic method to generate the $1/n$ expansion	218
B. Existence of the critical theory	219
C. Fixed point in the large $n$ -limit	220
D. Results of the calculations in the $1/n$ expansion	220
XI. Discussion of the General $n$ -Vector Model	221
A. The Hamiltonian density	224
B. Renormalization group equations	225
C. Critical exponents	228
D. Stability conditions	231
E. Anisotropic corrections to the equation of state for the symmetric fixed point	235
XII. Short Distance Expansions and Critical Phenomena	237
A. Short-distance expansion of an operator product	238
B. The renormalization group equation satisfied by the first Wilson coefficient	239
C. Next-to-leading terms in zero field above $T_c$	240
D. Next-to-leading terms in a field or below $T_c$	241
References	244

#### 4. The $1/n$ Expansion

SHANG-KENG MA

I. Introduction	250
A. Simplicity at large $n$	250
B. The model and the solution above the critical point	251
C. Historical note, summary of exponents	255
II. Renormalization Group, Scaling Fields and the Fixed Point	258
A. Renormalization group	259
B. Scaling fields	260
C. RG and scaling fields for $n \rightarrow \infty$ , critical surface	260
D. Transformation of the free energy	266
III. Scaling Variables, Scaling Dimensions and Correlation Functions	267
A. Transformation of random variables, scaling variables	267
B. Product of scaling variables	270
C. Correlation functions	270
D. Discussion	272

IV.	Nonzero Magnetization and the Equation of State at $n \rightarrow \infty$	273
A.	Transverse and longitudinal susceptibilities	273
B.	Solution below the critical point and the equation of state	274
V.	Calculation of Exponents to $O(1/n)$	276
A.	Perturbation theory	277
B.	Properties of $\Pi(k)$ and the dressed interaction	278
C.	Evaluation of $\eta$ and $\gamma$	280
D.	Evaluation of $\alpha$ and the Abe-Hikami anomaly	282
E.	Evaluation of $y_{\Delta\tau}$ and $y_{\tau_1}$	285
F.	Evaluation of $y_c$ , the cubic exponent	287
G.	Results	288
VI.	Discussion	288
A.	Other formulations	288
B.	Convergence and extrapolation	289
C.	Long-range force	290
D.	Concluding remarks	291
	References	291

## 5. The $\varepsilon$ -Expansion for Exponents and the Equation of State in Isotropic Systems

D. J. WALLACE

	Introduction	294
I.	General Formalism	295
A.	The $n$ -component field model	295
B.	Correlation functions	297
C.	Feynman rules	300
II.	The $\varepsilon$ -Expansion	304
A.	Résumé	304
B.	Perturbation theory in the critical region	305
C.	The $\varepsilon$ -expansion	308
D.	Results for $\eta$ and $\gamma$	314
E.	The irreducible diagram series	320
F.	Notes on evaluation of graphs	322
III.	Equation of State	326
A.	Introduction	326
B.	Setting up the calculation	327
C.	The $\varepsilon$ -expansion for the equation of state	333
D.	Parametric models	340
E.	Behaviour near the coexistence curve	343
IV.	Other Perturbations of the Hamiltonian	346
A.	General considerations	346
B.	Scaling variables of the Heisenberg fixed point	348
C.	Calculations and conclusions	350
	Acknowledgements	354
	References	354

## 6. Dependence of Universal Critical Behaviour on Symmetry and Range of Interaction

AMNON AHARONY

I.	Introduction	358
II.	The Recursion Relation Approach	359
	A. Reduced Hamiltonian: notations and normalizations	359
	B. Renormalization group iterations	363
	C. The Gaussian model	366
	D. Diagrammatic expansions	371
III.	Spin Anisotropies	376
	A. Crossover to a lower effective spin dimensionality	376
	B. Crossover to mean field behaviour as $n \rightarrow \infty$ (extreme anisotropy)	380
IV.	Long-Range Interactions and Dipolar Systems	382
	A. Isotropic long-range interactions	382
	B. Isotropic dipolar magnets; antiferromagnets	383
	C. Isotropic ferromagnets	387
	D. Anisotropic ferromagnets; X Y-like anisotropy	390
	E. Anisotropic Ising-like ferromagnets	392
V.	Generalized "Cubic" Systems: Various Effective Quartic Spin Interactions	394
	A. The competition between Heisenberg, Ising and cubic fixed points	394
	B. Cubic critical behaviour	398
	C. $n$ coupled $m$ -vector models	402
	D. The random $m$ -component magnet	406
	E. Metamagnetic, bicritical and tetracritical behaviour in antiferromagnets	410
	F. Compressible magnets	413
	G. Systems with larger "spin" dimensionality	416
VI.	Spatial Anisotropies	417
	A. Lattice anisotropies	417
	B. Semi-infinite systems	418
VII.	Discussion	419
	References	421

## 7. Renormalization Theory for Ising-like Spin Systems

TH. NIEMEIJER AND J. M. J. VAN LEEUWEN

Introduction	425	
I. Definitions	427	
II. General Theory	434	
	A. Eigenvalues and critical exponents	437
	B. Determination of the critical temperature	440
	C. Calculation of the free energy	440
	D. The spontaneous magnetization	447
III. Correlation Functions	450	
	A. Linear weight-factors $P(s', s)$	451
	B. Simplified treatment of $g(r)$	453
	C. General discussion of the correlation functions	457
	D. Connection between exponents and eigenvectors	462

IV. Computational Methods	463
A. Simple renormalization transformations on finite lattices	465
B. The cumulant approximation	467
C. The cluster approximation	470
V. Applications	473
A. The triangular lattice	474
B. The quadratic lattice	484
VI. Discussion	492
References	499
Appendix A	500
Appendix B	502

## 8. Renormalization Group Approach to Critical Phenomena

C. DI CASTRO AND G. JONA-LASINIO

I. Introduction	508
II. Scaling	511
III. Block Variables and the Kadanoff-Wilson Approach	514
A. Block variable transformation	514
B. Scaling properties at the critical point	516
C. Linearized group transformation	518
D. Properties of approach to the critical point	520
IV. Functional Form of Many-Body Theory	522
A. Introductory remarks	522
B. The model	523
C. Functional equations	526
V. The Field Theoretic Renormalization Group	528
A. Group equations	528
B. Scaling	533
VI. $\epsilon$ -Expansion and Approximate Treatment of Scaling Variables and Cross-over Effects	537
A. $\epsilon$ -expansion	537
B. Scaling variables and cross-over effects	538
VII. Formal Equivalence of Kadanoff-Wilson and Field Theoretic Renormalization Groups	540
VIII. Generalized and Non-Linear Transformations	544
References	551
Appendix A. A Connection with Probability Theory	552
Appendix B. Summary of Diagrammatic Expansions	554
Appendix C. Relation to the Callan-Symanzik Equation	555
Author Index	559
Subject Index	565