Contents

	Preface	xiii
	Introduction	xv
PART	I ESSENTIALS OF NONLINEAR DYNAMICS	1
1	Nonlinear mappings	1
1.1	Maps and trajectories	1
1.2	Attractors	2
1.3	The logistic map	4
1.4	Maps of Kaplan-Yorke type	11
1.5	The Hénon map	14
1.6	The standard map	16
2	Probability in the theory of chaotic systems	20
2.1	Relative frequencies and probability	20
2.2	Invariant densities	22
2.3	Topological conjugation	28
3	Symbolic dynamics	32
3.1	Partitions of the phase space	32
3.2	The binary shift map	33
3.3	Generating partition	35
3.4	Symbolic dynamics of the logistic map	37
3.5	Cylinders	39
3.6	Symbolic stochastic processes	40

viii Contents

PART	II ESSENTIALS OF INFORMATION THEORY AND THERMODYNAMICS	44
4	The Shannon information	44
4.1 4.2 *4.3	Bit-numbers The Shannon information measure The Khinchin axioms	44 46 47
5	Further information measures	50
5.1 5.2 *5.3	The Rényi information The information gain General properties of the Rényi information	50 51 53
6	Generalized canonical distributions	56
6.1 6.2 6.3 *6.4	The maximum entropy principle The thermodynamic equilibrium ensembles The principle of minimum free energy Arbitrary a priori probabilities	56 59 61 63
7	The fundamental thermodynamic relations	65
7.1 7.2 *7.3 7.4 7.5	The Legendre transformation Gibbs' fundamental equation The Gibbs-Duhem equation Susceptibilities and fluctuations Phase transitions	65 68 71 73 75
8	Spin systems	78
8.1 8.2 8.3	Various types of spin models Phase transitions of spin systems Equivalence between one-dimensional spin systems and symbolic stochastic processes The transfer matrix method	78 82 83 85
PART	III THERMOSTATISTICS OF MULTIFRACTALS	88
9	Escort distributions	88
9.1 9.2 *9.3	Temperature in chaos theory Two or more intensities Escort distributions for general test functions	88 90 91

Contents	1X

10	Fractals	94
10.1	Simple examples of fractals	94
10.2	The fractal dimension	97
10.3	The Hausdorff dimension	100
10.4	Mandelbrot and Julia sets	102
*10.5	Iterated function systems	109
11	Multifractals	114
11.1	The grid of boxes of equal size	114
11.2	The Rényi dimensions	115
11.3	Thermodynamic relations in the limit of box size going	
11.4	to zero	117
11.4	Definition of the Rényi dimensions for cells of variable size	123
*12	Bit-cumulants in multifractal statistics	127
12.1	Moments and cumulants of bit-numbers	127
12.2	The bit-variance in thermodynamics	129
12.3	The sensitivity to correlations	131
12.4	Heat capacity of a fractal distribution	132
*13	Thermodynamics of finite volume	136
13.1	Boxes of finite size	136
13.2	Thermodynamic potentials for finite volume	139
13.3	An analytically solvable example	143
PART	IV DYNAMICAL ANALYSIS OF CHAOTIC	
PAKI	SYSTEMS	146
	Section (Control of Control of Co	
14	Statistics of dynamical symbol sequences	146
14.1	The Kolmogorov-Sinai entropy	146
14.2	The Rényi entropies	149
14.3	Equivalence with spin systems	155
14.4	Spectra of dynamical scaling indices	156
15	Expansion rate and information loss	158
15.1	Expansion of one-dimensional systems	158
15.2	The information loss	160
*15.3	The variance of the loss	161
15.4	Spectra of local Liapunov exponents	164
15.5	Liapunov exponents for higher-dimensional systems	167
15.6	Stable and unstable manifolds	172

x Contents

16	The topological pressure	178
16.1	Definition of the topological pressure	178
16.2	Length scale interpretation	182
16.3	Examples	184
*16.4	Extension to higher dimensions	187
*16.5	Topological pressure for arbitrary test functions	188
17	Transfer operator methods	190
17.1	The Perron-Frobenius operator	190
17.2	Invariant densities as fixed points of the Perron-Frobenius operator	193
17.3	Spectrum of the Perron-Frobenius operator	196
17.4	Generalized Perron-Frobenius operator and topological	197
*17.5	pressure Connection with the transfer matrix method of classical	197
17.3	statistical mechanics	199
18	Repellers and escape	204
18.1	Repellers	204
18.2	Escape rate	207
*18.3	The Feigenbaum attractor as a repeller	208
PART	V ADVANCED THERMODYNAMICS	211
19	Thermodynamics of expanding maps	211
19.1	A variational principle for the topological pressure	211
19.2	Gibbs measures and SRB measures	214
19.3	Relations between topological pressure and the Rényi entropies	218
19.4	Relations between topological pressure and the generalized Liapunov exponents	221
19.5	Relations between topological pressure and the Rényi	
	dimensions	222
20	Thermodynamics with several intensive parameters	226
20.1	The pressure ensemble	226
*20.2		220
	Equation of state of a chaotic system	229
20.3	The grand canonical ensemble	233
20.3 20.4 *20.5		

Contents		XI
21	Phase transitions	243
21.1	Static phase transitions	243
21.2	Dynamical phase transitions	248
21.3	Expansion phase transitions	251
21.4	Bivariate phase transitions	254
21.5	Phase transitions with respect to the volume	256
21.6	External phase transitions of second order	260
21.7	Renormalization group approach	262
21.8	External phase transitions of first order	267
	References	273
	Index	282