Contents

Foreword xi

Part I. The Delta Function and its Applications 1
1. Introduction 1
2. The Dirac delta function 3
   2.1. The delta function and the set of orthonormal functions 3
   2.2. The delta function and the Fourier expansion 5
   2.3. Generalization of some of the properties of a continuous function to the delta function 9
   2.4. Some properties of the delta function 11
3. Solution of linear equations with the aid of the delta function 12
   3.1. The delta function in a space of several dimensions 12
   3.2. Green's function 13
4. Solution of Poisson's equation 14
5. Solution of d'Alembert's equation 15

Part II. Classical Electrodynamics 19
6. Basic equations of classical electrodynamics 19
   6.1. The Maxwell–Lorentz equations 19
   6.2. Integration of the Maxwell–Lorentz equations 20
7. Maxwell–Lorentz equations in four-dimensional form 22
   7.1. Covariant and contravariant vectors and tensors 22
   7.2. Relativistically covariant form of the Maxwell–Lorentz equations 24
   7.3. Action 26
   7.4. The Lagrange and Hamilton equations in the relativistic theory of the electron 29
   7.5. The Hamilton–Jacobi equation in relativistic theory 32
8. Motion of an electron in a constant uniform magnetic field 34
9. The radiation problem 40
   9.1. The energy of an electromagnetic field in a vacuum and the Poynting theorem 40
   9.2. Maxwell's equations in a medium 41
   9.3. Boundary conditions 44

9.4. Čerenkov effect 45
9.5. Equipotential surfaces during the motion of an electron at constant velocity 50
9.6. Emission by a harmonic oscillator 52
9.7. Polarization of the radiation emitted by the harmonic oscillator 55
9.8. Total emission 56
9.9. Dipole and quadrupole emission by the harmonic oscillator .. 56
10. Synchrotron radiation 57
10.1. Basic equations 58
10.2. Derivation of Schott's formula 60
10.3. Polarization properties of radiation emitted during the motion of an electron on a helix 62
10.4. Angular distribution of radiation 66
10.5. Spectral distribution 68
10.6. Spectral distribution for nonrelativistic circular motion 70
10.7. Asymptotic representation of high-order Bessel functions 71
10.8. Ultra-relativistic circular motion 74
10.9. Spectral distribution (approximate formulae) 76
10.10. Angular distribution of the radiated intensity (approximate formulae) 78
10.11. Approximate formulae for helical motion 80
10.12. Experimental verification of the predictions of the classical theory of synchrotron radiation 82
11. The classical Dirac–Lorentz equation for the point electron 87
11.1. Derivation of the equation 87
11.2. Radiated energy 91
11.3. Hyperbolic motion 93
11.4. Rectilinear motion 94
11.5. Hyperbolic motion in a finite interval of time 97
11.6. Motion of an electron in a constant uniform magnetic field with allowance for the frictional force 99
11.7. Motion on a helix with compensation of radiative energy loss 101
11.8. Motion of an electron in a focusing magnetic field with allowance for radiative reaction 104

Part III. Relativistic Quantum Theory 109
12. Variational principles for the electromagnetic field in the absence of charges 109
12.1. Energy and angular momentum tensors 109
12.2. Integral form of the conservation laws 113
12.3. Solution of d'Alembert's equation 114
12.4. Polarization properties 115
12.5. Quantization of the electromagnetic field 117
12.6. Quantization of the electromagnetic field in the general case .. 122

13. The scalar relativistic wave equation of Klein and Gordon 126
13.1. Derivation of the equation 126
13.2. Transformation properties of the wavefunction 127
13.3. Charge and current densities 128
13.4. Klein–Gordon equation for a particle in the electromagnetic field 129
13.5. Variational methods 130
13.6. Free motion 132
13.7. Quantization of the free field 134
14. Dirac's equation 136
14.1. Linearization of the energy operator: Dirac's matrices 136
14.2. Dirac's equation: charge and current density 139
14.3. Transformation properties of the wavefunction 141
14.4. Dirac equation in covariant form 143
14.5. Tensor dimensionality of Dirac matrices 144
15. Variational methods 147
15.1. The Lagrange function 147
15.2. Energy tensor 148
15.3. Orbital and spin angular momentum tensors 149
15.4. Second quantization of the electron–positron field 152
15.5. Wave equation for the positron: charge conjugation 155
15.6. Lüders–Pauli theorem 156
16. Polarization operators: spin states of the electron 157
16.1. Polarization operators for the free motion of the electron 157
16.2. Wavefunction for a freely moving electron 163
16.3. Spin properties of the free electron 165
16.4. Polarization operators in the presence of the electromagnetic field 168
16.5. Single-particle approximation: Bargman–Michel–Telegdi equation 170
17. The centrally symmetric field 174
17.1. Kepler's problem for the Klein–Gordon equation 174
17.2. Dirac's theory of the motion of an electron in a central field .. 180
17.3. Properties of the total angular momentum: spherical spinors . 181
17.4. Dirac's theory of the motion of an electron in a Coulomb field 185
17.5. Energy spectrum 189
18. Theory of radiation 191
18.1. Theory of transient processes 191
18.2. Polarization effects 195
18.3. Spontaneous and stimulated transitions 196
18.4. Dipole radiation 198
18.5. Magnetic and quadropole radiation 200

Part IV. Quantum Effects in Synchrotron Radiation 203
19. Motion of an electron in a constant uniform magnetic field 203
19.1. The Klein–Gordon equation in cylindrical coordinates 203
19.2. Solution of the Klein–Gordon equation in Cartesian coordinates 207
19.3. Motion of an electron in a constant magnetic field: Dirac theory in terms of cylindrical coordinates 208
19.4. Solution of the Dirac equation in terms of Cartesian coordinates 215
20. Quantum theory of emission by an electron moving in a constant uniform magnetic field 216
20.1. General formulae for spontaneous emission with allowance for polarization 216
20.2. Quasiquantum approximation 219
20.3. Effect of quantum fluctuations on radial motion 225
20.4. Effect of synchrotron radiation on vertical motion 228
20.5. Effect of quantum mechanical fluctuations on the operation of a cyclic accelerator 230
21. Quantum mechanical effects 234
21.1. General formulae 234
21.2. Asymptotic representation of Laguerre functions 236
21.3. Effect of the "transverse" polarization of the electron spin (self-polarization effect) 240
21.4. Quantum mechanical effects in radiation intensity 245
22. Mutual transitions between photons and electron–positron pairs in a magnetic field 248
22.1. General ideas 248
22.2. Single-photon pair production in a magnetic field 249
22.3. Single-photon pair annihilation 257
23. Vacuum magnetic moment of the electron 258
23.1. General ideas 258
23.2. Radiative corrections to the Dirac equation 259
23.3. The case of small n 262
23.4. The case of large n 262
24. Scattering of a circularly polarized plane electromagnetic wave by an electron 264
24.1. Nonlinear classical theory 264
24.2. Theory of scattering 266
24.3. Quantum theory of radiation 269
25. Stimulated transitions in the theory of synchrotron radiation 273
26. Effect of synchrotron radiation on the motion of electrons in an inhomogeneous magnetic field 277
26.1. Motion of electrons in an axially symmetric magnetic field ... 277
26.2. General formulae for the matrix elements and transition probabilities for synchrotron radiation 282

26.3. Effect of synchrotron radiation on energy losses and betatron oscillations 284
26.4. Motion of electrons in an axially symmetric focusing field (Dirac's theory) 291
27. Experimental study of quantal effects in synchrotron radiation 293
27.1. Observation of quantal fluctuations in the particle orbit (orbit broadening effect) 293
27.2. Observation of radiative polarization of electrons and positrons 296

References 301

Index 307