CONTENTS

PAGE

PREFACE v

CHAPTER I

THE FIELD EQUATIONS

MAXWELL'S EQUATIONS 1

1.1 The Field Vectors 1
1.2 Charge and Current 2
1.3 Divergence of the Field Vectors 6
1.4 Integral Form of the Field Equations 6

MACROSCOPIC PROPERTIES OF MATTER 10

1.5 The Inductive Capacities ϵ and μ 10
1.6 Electric and Magnetic Polarization 11
1.7 Conducting Media 13

UNITS AND DIMENSIONS 16

1.8 M.K.S. or Giorgi System 16

THE ELECTROMAGNETIC POTENTIALS 23

1.9 Vector and Scalar Potentials 23
1.10 Conducting Media 26
1.11 Hertz Vectors, or Polarization Potentials 28
1.12 Complex Field Vectors and Potentials 32

BOUNDARY CONDITIONS 34

1.13 Discontinuities in the Field Vectors 34

COORDINATE SYSTEMS 38

1.14 Unitary and Reciprocal Vectors 38
1.15 Differential Operators 44
1.16 Orthogonal Systems 47
1.17 Field Equations in General Orthogonal Coordinates 50
1.18 Properties of Some Elementary Systems 51

THE FIELD TENSORS 59

1.19 Orthogonal Transformations and Their Invariants 59
1.20 Elements of Tensor Analysis 64
1.21 Space-time Symmetry of the Field Equations 69
1.22 The Lorentz Transformation 74
1.23 Transformation of the Field Vectors to Moving Systems 78

PAGE

CHAPTER II

STRESS AND ENERGY

STRESS AND STRAIN IN ELASTIC MEDIA 83
2.1 Elastic Stress Tensor 83
2.2 Analysis of Strain 87
2.3 Elastic Energy and the Relations of Stress to Strain 93

ELECTROMAGNETIC FORCES ON CHARGES AND CURRENTS 96
2.4 Definition of the Vectors **E** and **B** 96
2.5 Electromagnetic Stress Tensor in Free Space 97
2.6 Electromagnetic Momentum 103

ELECTROSTATIC ENERGY 104
2.7 Electrostatic Energy as a Function of Charge Density 104
2.8 Electrostatic Energy as a Function of Field Intensity 107
2.9 A Theorem on Vector Fields 111
2.10 Energy of a Dielectric Body in an Electrostatic Field 112
2.11 Thomson's Theorem 114
2.12 Earnshaw's Theorem 116
2.13 Theorem on the Energy of Uncharged Conductors 117

MAGNETOSTATIC ENERGY 118
2.14 Magnetic Energy of Stationary Currents 118
2.15 Magnetic Energy as a Function of Field Intensity 123
2.16 Ferromagnetic Materials 125
2.17 Energy of a Magnetic Body in a Magnetostatic Field 126
2.18 Potential Energy of a Permanent Magnet 129

ENERGY FLOW 131
2.19 Poynting's Theorem 131
2.20 Complex Poynting Vector 135

FORCES ON A DIELECTRIC IN AN ELECTROSTATIC FIELD 137
2.21 Body Forces in Fluids 137
2.22 Body Forces in Solids 140
2.23 The Stress Tensor 146
2.24 Surfaces of Discontinuity 147
2.25 Electrostriction 149
2.26 Force on a Body Immersed in a Fluid 151

FORCES IN THE MAGNETOSTATIC FIELD 153
2.27 Nonferromagnetic Materials 153
2.28 Ferromagnetic Materials 155

FORCES IN THE ELECTROMAGNETIC FIELD 156
2.29 Force on a Body Immersed in a Fluid 156

CHAPTER III

THE ELECTROSTATIC FIELD

GENERAL PROPERTIES OF AN ELECTROSTATIC FIELD 160
3.1 Equations of Field and Potential 160
3.2 Boundary Conditions 163

PAGE

CALCULATION OF THE FIELD FROM THE CHARGE DISTRIBUTION 165

3.3 Green's Theorem 165
3.4 Integration of Poisson's Equation 166
3.5 Behavior at Infinity 167
3.6 Coulomb Field 169
3.7 Convergence of Integrals 170

EXPANSION OF THE POTENTIAL IN SPHERICAL HARMONICS 172

3.8 Axial Distributions of Charge 172
3.9 The Dipole 175
3.10 Axial Multipoles 176
3.11 Arbitrary Distributions of Charge 178
3.12 General Theory of Multipoles 179

DIELECTRIC POLARIZATION 183

3.13 Interpretation of the Vectors **P** and Π 183

DISCONTINUITIES OF INTEGRALS OCCURRING IN POTENTIAL THEORY 185

3.14 Volume Distributions of Charge and Dipole Moment 185
3.15 Single-layer Charge Distributions 187
3.16 Double-layer Distributions 188
3.17 Interpretation of Green's Theorem 192
3.18 Images 193

BOUNDARY-VALUE PROBLEMS 194

3.19 Formulation of Electrostatic Problems 194
3.20 Uniqueness of Solution 196
3.21 Solution of Laplace's Equation 197

PROBLEM OF THE SPHERE 201

3.22 Conducting Sphere in Field of a Point Charge 201
3.23 Dielectric Sphere in Field of a Point Charge 204
3.24 Sphere in a Parallel Field 205

PROBLEM OF THE ELLIPSOID 207

3.25 Free Charge on a Conducting Ellipsoid 207
3.26 Conducting Ellipsoid in a Parallel Field 209
3.27 Dielectric Ellipsoid in a Parallel Field 211
3.28 Cavity Definitions of **E** and **D** 213
3.29 Torque Exerted on an Ellipsoid 215

PROBLEMS 217

CHAPTER IV

THE MAGNETOSTATIC FIELD

GENERAL PROPERTIES OF A MAGNETOSTATIC FIELD 225

4.1 Field Equations and the Vector Potential 225
4.2 Scalar Potential 226
4.3 Poisson's Analysis 228

CALCULATION OF THE FIELD OF A CURRENT DISTRIBUTION 230

4.4 Biot-Savart Law 230
4.5 Expansion of the Vector Potential 233

PAGE
4.6 The Magnetic Dipole . . . 236
4.7 Magnetic Shells . . . 237

A DIGRESSION ON UNITS AND DIMENSIONS . . . 238
4.8 Fundamental Systems . . . 238
4.9 Coulomb's Law for Magnetic Matter . . . 241

MAGNETIC POLARIZATION . . . 242
4.10 Equivalent Current Distributions . . . 242
4.11 Field of Magnetized Rods and Spheres . . . 243

DISCONTINUITIES OF THE VECTORS **A** AND **B** . . . 245
4.12 Surface Distributions of Current . . . 245
4.13 Surface Distributions of Magnetic Moment . . . 247

INTEGRATION OF THE EQUATION $\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$ . . . 250
4.14 Vector Analogue of Green's Theorem . . . 250
4.15 Application to the Vector Potential . . . 250

BOUNDARY-VALUE PROBLEMS . . . 254
4.16 Formulation of the Magnetostatic Problem . . . 254
4.17 Uniqueness of Solution . . . 256

PROBLEM OF THE ELLIPSOID . . . 257
4.18 Field of a Uniformly Magnetized Ellipsoid . . . 257
4.19 Magnetic Ellipsoid in a Parallel Field . . . 258

CYLINDER IN A PARALLEL FIELD . . . 258
4.20 Calculation of the Field . . . 258
4.21 Force Exerted on the Cylinder . . . 261
PROBLEMS . . . 262

CHAPTER V

PLANE WAVES IN UNBOUNDED, ISOTROPIC MEDIA

PROPAGATION OF PLANE WAVES . . . 268
5.1 Equations of a One-dimensional Field . . . 268
5.2 Plane Waves Harmonic in Time . . . 273
5.3 Plane Waves Harmonic in Space . . . 278
5.4 Polarization . . . 279
5.5 Energy Flow . . . 281
5.6 Impedance . . . 282

GENERAL SOLUTIONS OF THE ONE-DIMENSIONAL WAVE EQUATION . . . 284
5.7 Elements of Fourier Analysis . . . 285
5.8 General Solution of the One-dimensional Wave Equation in a Nondissipative Medium . . . 292
5.9 Dissipative Medium; Prescribed Distribution in Time . . . 297
5.10 Dissipative Medium; Prescribed Distribution in Space . . . 301
5.11 Discussion of a Numerical Example . . . 304
5.12 Elementary Theory of the Laplace Transformation . . . 309
5.13 Application of the Laplace Transformation to Maxwell's Equations . . . 318

PAGE

DISPERSION 321
5.14 Dispersion in Dielectrics 321
5.15 Dispersion in Metals 325
5.16 Propagation in an Ionized Atmosphere 327

VELOCITIES OF PROPAGATION 330
5.17 Group Velocity 330
5.18 Wave-front and Signal Velocities 333

PROBLEMS 340

CHAPTER VI
CYLINDRICAL WAVES

EQUATIONS OF A CYLINDRICAL FIELD 349
6.1 Representation by Hertz Vectors 349
6.2 Scalar and Vector Potentials 351
6.3 Impedances of Harmonic Cylindrical Fields 354

WAVE FUNCTIONS OF THE CIRCULAR CYLINDER 355
6.4 Elementary Waves 355
6.5 Properties of the Functions $Z_p(\rho)$ 357
6.6 The Field of Circularly Cylindrical Wave Functions 360

INTEGRAL REPRESENTATIONS OF WAVE FUNCTIONS 361
6.7 Construction from Plane Wave Solutions 361
6.8 Integral Representations of the Functions $Z_n(\rho)$ 364
6.9 Fourier-Bessel Integrals 369
6.10 Representation of a Plane Wave 371
6.11 The Addition Theorem for Circularly Cylindrical Waves 372

WAVE FUNCTIONS OF THE ELLIPTIC CYLINDER 375
6.12 Elementary Waves 375
6.13 Integral Representations 380
6.14 Expansion of Plane and Circular Waves 384

PROBLEMS 387

CHAPTER VII
SPHERICAL WAVES

THE VECTOR WAVE EQUATION 392
7.1 A Fundamental Set of Solutions 392
7.2 Application to Cylindrical Coordinates 395

THE SCALAR WAVE EQUATION IN SPHERICAL COORDINATES 399
7.3 Elementary Spherical Waves 399
7.4 Properties of the Radial Functions 404
7.5 Addition Theorem for the Legendre Polynomials 406
7.6 Expansion of Plane Waves 408
7.7 Integral Representations 409
7.8 A Fourier-Bessel Integral 411
7.9 Expansion of a Cylindrical Wave Function 412
7.10 Addition Theorem for $z_0(kR)$ 413

PAGE

THE VECTOR WAVE EQUATION IN SPHERICAL COORDINATES 414
7.11 Spherical Vector Wave Functions 414
7.12 Integral Representations 416
7.13 Orthogonality 417
7.14 Expansion of a Vector Plane Wave 418
PROBLEMS 420

CHAPTER VIII

RADIATION

THE INHOMOGENEOUS SCALAR WAVE EQUATION 424
8.1 Kirchhoff Method of Integration 424
8.2 Retarded Potentials 428
8.3 Retarded Hertz Vector 430
A MULTIPOLE EXPANSION 431
8.4 Definition of the Moments 431
8.5 Electric Dipole 434
8.6 Magnetic Dipole 437
RADIATION THEORY OF LINEAR ANTENNA SYSTEMS 438
8.7 Radiation Field of a Single Linear Oscillator 438
8.8 Radiation Due to Traveling Waves 445
8.9 Suppression of Alternate Phases 446
8.10 Directional Arrays 448
8.11 Exact Calculation of the Field of a Linear Oscillator 454
8.12 Radiation Resistance by the E.M.F. Method 457
THE KIRCHHOFF-HUYGENS PRINCIPLE 460
8.13 Scalar Wave Functions 460
8.14 Direct Integration of the Field Equations 464
8.15 Discontinuous Surface Distributions 468
FOUR-DIMENSIONAL FORMULATION OF THE RADIATION PROBLEM 470
8.16 Integration of the Wave Equation 470
8.17 Field of a Moving Point Charge 473
PROBLEMS 477

CHAPTER IX

BOUNDARY-VALUE PROBLEMS

GENERAL THEOREMS 483
9.1 Boundary Conditions 483
9.2 Uniqueness of Solution 486
9.3 Electrodynamic Similitude 488
REFLECTION AND REFRACTION AT A PLANE SURFACE 490
9.4 Snell's Laws 490
9.5 Fresnel's Equations 492
9.6 Dielectric Media 494
9.7 Total Reflection 497
9.8 Refraction in a Conducting Medium 500
9.9 Reflection at a Conducting Surface 505

PAGE

PLANE SHEETS 511
9.10 Reflection and Transmission Coefficients 511
9.11 Application to Dielectric Media 513
9.12 Absorbing Layers 515

SURFACE WAVES 516
9.13 Complex Angles of Incidence 516
9.14 Skin Effect 520

PROPAGATION ALONG A CIRCULAR CYLINDER 524
9.15 Natural Modes 524
9.16 Conductor Embedded in a Dielectric 527
9.17 Further Discussion of the Principal Wave 531
9.18 Waves in Hollow Pipes 537

COAXIAL LINES 545
9.19 Propagation Constant 545
9.20 Infinite Conductivity 548
9.21 Finite Conductivity 551

OSCILLATIONS OF A SPHERE 554
9.22 Natural Modes 554
9.23 Oscillations of a Conducting Sphere 558
9.24 Oscillations in a Spherical Cavity 560

DIFFRACTION OF A PLANE WAVE BY A SPHERE 563
9.25 Expansion of the Diffracted Field 563
9.26 Total Radiation 568
9.27 Limiting Cases 570

EFFECT OF THE EARTH ON THE PROPAGATION OF RADIO WAVES 573
9.28 Sommerfeld Solution 573
9.29 Weyl Solution 577
9.30 van der Pol Solution 582
9.31 Approximation of the Integrals 583

PROBLEMS 588

APPENDIX I

A. NUMERICAL VALUES OF FUNDAMENTAL CONSTANTS 601
B. DIMENSIONS OF ELECTROMAGNETIC QUANTITIES 601
C. CONVERSION TABLES 602

APPENDIX II

FORMULAS FROM VECTOR ANALYSIS 604

APPENDIX III

CONDUCTIVITY OF VARIOUS MATERIALS 605
SPECIFIC INDUCTIVE CAPACITY OF DIELECTRICS 606

APPENDIX IV

ASSOCIATED LEGENDRE FUNCTIONS 608

INDEX 609