

CONTENTS

	PAGE
PREFACE	xi
Part I: Introduction	
CHAPTER 1 INTRODUCTION	3
CHAPTER 2 BASIC FIELD THEORY	7
2.1 Electric Fields	7
2.1.1 The Electrostatic Field Vectors	7
2.1.2 Electric Potential	8
2.1.3 Potential Function and Flux Function	10
2.1.4 Capacitance	12
2.1.5 Laplace's and Poisson's Equations	12
2.1.6 Principle of Superposition	15
2.1.7 Electric Fields of Currents	15
2.2 Magnetic Fields	16
2.2.1 The Field of Magnetic Poles	16
2.2.2 The Magnetic Field of Line Currents	17
2.2.3 The Magnetic Field of Distributed Currents	18
2.2.4 Inductance	22
2.3 Boundary Conditions	23
2.4 Conjugate Functions	25
2.4.1 Laplace's Equation	25
2.4.2 Cauchy-Riemann Equations	27
2.4.3 Flux and Potential Functions as Conjugate Functions	28
2.4.4 Simple Examples of the Use of Conjugate Functions	29
2.5 Equivalent Pole and Charge Distributions	30
2.6 Forces	32
2.6.1 Line Sources	32
2.6.2 Distributed Sources	32
2.6.3 Total Force Acting on a Boundary	32
2.6.4 Force Distribution over a Boundary	33
References	33

Part II: Direct methods

CHAPTER 3 IMAGES	37
3.1 Introduction	37
3.2 Plane Boundaries	38
3.2.1 Single Plane Boundary	38
3.2.2 Parallel Plane Boundaries	41
3.2.3 Intersecting Plane Boundaries	42
3.2.4 Inductance of Parallel Bus-bars near an Iron Surface	45
3.3 Circular Boundaries	46
3.3.1 Charge or Current near a Circular Boundary	46
3.3.2 Doublets: Circular Cylinder in a Uniform Field	48
3.4 General Considerations	54
References	57
 CHAPTER 4 THE SOLUTION OF LAPLACE'S EQUATION BY SEPARATION OF THE VARIABLES	 59
4.1 Introduction	59
4.2 Circular Boundaries	60
4.2.1 The Solution of Laplace's Equation in Circular-cylinder Coordinates	60
4.2.2 Iron Cylinder Influenced by a Current	63
4.2.3 The Screening Effect of a Permeable Cylinder	65
4.2.4 The Force between Rotor and Stator Conductors in a Cylindrical Machine	68
4.2.5 Specified Distributions of Potential or Potential Gradient on the Perimeter of a Circular Boundary	71
4.3 Rectangular Boundaries	72
4.3.1 Solution of Laplace's Equation in Cartesian Coordinates	72
4.3.2 The Semi-infinite Strip and the Rectangle	74
4.3.3 Pole Profile in the Inductor Alternator for a Sinusoidal Flux Distribution	77
4.4 Conclusions	80
References	81
 CHAPTER 5 THE SOLUTION OF POISSON'S EQUATION: MAGNETIC FIELDS OF DISTRIBUTED CURRENT	 83
5.1 Introduction	83
5.2 Non-magnetic Conductors in Air	85
5.2.1 The Method: Vector Potential of a Line Current	85
5.2.2 The Field of a Rectangular Bus-bar	85
5.2.3 The Force between Parallel Rectangular Bus-bars	88

CONTENTS

vii

5.3 The Field inside Infinitely Permeable Conductors in Air	91
5.3.1 General Considerations	91
5.3.2 The Field inside a Highly Permeable Rectangular Conductor	92
5.4 Simple Boundaries: Use of the Image Method	94
5.5 The Treatment of Boundaries using Single Fourier Series: Rogowski's Method	95
5.5.1 Rectangular Conductors in an Infinite, Parallel Air Gap	95
5.5.2 Finite Boundaries: Rectangular Conductor in a Slot	99
5.5.3 Scope of the Method	102
5.6 The Treatment of Boundaries using Double Fourier Series: Roth's Method	103
5.6.1 The Method	103
5.6.2 The Forces on, and the Inductance of, a Transformer Winding	107
5.6.3 Conductor in Slot: Calculation of Inductance	109
5.6.4 Scope of the Method	110
References	112

Part III: Transformation methods

CHAPTER 6 INTRODUCTION TO CONFORMAL TRANSFORMATION	117
6.1 Conformal Transformation and Conjugate Functions	117
6.1.1 Conformal Transformation	117
6.1.2 The Solution of Laplace's Equation	120
6.1.3 The Logarithmic Function	122
6.2 Classes of Solvable Problems	123
6.3 General Considerations	124
6.3.1 Choice of Origin	125
6.3.2 Multiple Transformations	125
6.3.3 Field Maps	126
6.3.4 Scale Relationship between Planes	126
6.3.5 Conservation of Flux and Potential	126
6.3.6 Field Strength	127
6.4 The Determination of Transformation Equations	128
References	128
CHAPTER 7 CURVED BOUNDARIES	129
7.1 The Bilinear Transformation	129
7.1.1 Mapping Properties	130
7.1.2 The Cross-ratio	134
7.1.3 The Magnetic Field of Currents inside an Infinitely Permeable Tube	135
7.1.4 The Capacitance of and the Voltage Gradient between Two Cylindrical Conductors	136
7.2 The Simple Joukowski Transformation	140
7.2.1 The Transformation	140
7.2.2 Flow round a Circular Hole	141
7.2.3 Permeable Cylinder Influenced by a Line Current	142

7.3 Curves Expressible Parametrically: General Series Transformations	145
7.3.1 The Method	145
7.3.2 The Field outside a Charged, Conducting Boundary of Elliptical Shape	146
7.3.3 General Series Transformations	147
7.3.4 Field Solutions	148
References	149
 CHAPTER 8 POLYGONAL BOUNDARIES	 151
8.1 Introduction	151
8.2 Transformation of the Upper Half Plane into the Interior of a Polygon	152
8.2.1 The Transformation	152
8.2.2 Polygons with Two Vertices	154
8.2.3 Parallel Plate Condenser: Rogowski Electrode	156
8.2.4 The Choice of Corresponding Points	160
8.2.5 Scale Relationship between Planes	163
8.2.6 The Field of a Current in a Slot	165
8.2.7 Negative Vertex Angles	169
8.2.8 The Forces between the Armature and Magnet of a Contactor	171
8.2.9 A Simple Electrostatic Lens	175
8.3 Transformation of the Upper Half Plane into the Region Exterior to a Polygon	177
8.3.1 The Transformation	177
8.3.2 The Field of a Charged, Conducting Plate	179
8.4 Transformations from a Circular to a Polygonal Boundary	182
8.4.1 The Transformation Equations	182
8.4.2 The Field of a Line Current and a Permeable Plate of Finite Cross-section	184
8.5 Classification of Integrals	186
References	187
 CHAPTER 9 THE USE OF ELLIPTIC FUNCTIONS	 189
9.1 Introduction	189
9.2 Elliptic Integrals and Functions	190
9.2.1 The Elliptic Integral of the First Kind	190
9.2.2 The Principal Jacobian Elliptic Functions	191
9.2.3 The Elliptic Integral of the Second Kind	191
9.2.4 Two Finite Charged Plates	192
9.2.5 Elliptic Integrals of the Third Kind	195
9.3 The Field outside a Charged Rectangular Conductor	196
9.3.1 The Transformation from an Infinite, Straight Line	197
9.3.2 The Transformation from a Circular Boundary	205
9.4 The Field in a Slot of Finite Depth	208
9.5 Conclusions	213
References	214

CHAPTER 10 GENERAL CONSIDERATIONS	217
10.1 Introduction	217
10.2 Field Sources	217
10.2.1 Infinite Boundaries	218
10.2.2 Finite Boundaries	222
10.2.3 Distributed Sources	223
10.3 Curved Boundaries	224
10.3.1 Rounded Corners	224
10.3.2 Curvilinear Polygons	226
10.4 Angles Not Multiples of $\pi/2$	227
10.4.1 Two-vertex Problems	227
10.5 Numerical Methods	228
10.5.1 Numerical Integration of the Function $f(t)$	229
10.5.2 Solution of the Implicit Equations	231
10.5.3 The Centring Force due to Displaced Ventilating Ducts in Rotating Machines	232
10.6 Non-equipotential Boundaries	235
10.6.1 Boundary Value Problems of the First Kind	235
10.6.2 Boundary Value Problems of the Second and Mixed Kinds	237
References	237

Part IV: Numerical methods

CHAPTER 11 FINITE-DIFFERENCE METHODS	241
11.1 Introduction	241
11.2 Finite-difference Representation	242
11.2.1 Regular Distributions of Field Points	242
11.2.2 Basic Equations for the Square and Rectangular Meshes	244
11.2.3 Field Problem as a Set of Simultaneous Equations	246
11.3 Hand Computation: Relaxation	248
11.3.1 Introduction	248
11.3.2 The Basic Method	249
11.3.3 Accelerating Processes	252
11.3.4 Practical Aspects	255
11.3.5 The Use of Point Values of Potential: Capacitance of a Three-core Rectangular Cable	256
11.4 Machine Computation: Iteration	258
11.4.1 Introduction	258
11.4.2 Basic Considerations and Methods	258
11.4.3 The Successive Over-relaxation Method	260
11.4.4 Current Flow in an I-section Conductor	265
11.4.5 Other Rapidly Convergent Methods	266
11.4.6 A Special Technique	267

11.5 Gradient Boundary Conditions	268
11.5.1 Introduction	268
11.5.2 Boundaries Coincident with Nodes	268
11.5.3 Boundaries Non-coincident with Nodes	273
11.5.4 Lines of Symmetry	276
11.5.5 Two Examples	277
11.6 Errors	279
11.6.1 Introduction	279
11.6.2 Mesh Error	279
11.6.3 Computational Errors	283
11.7 Conclusions	284
References	285
 CHAPTER 12 THE MONTE CARLO METHOD	 289
12.1 Introduction	289
12.2 The Method	289
12.3 Example	291
12.4 Some General Points	292
References	292
 Appendixes	
APPENDIX I The Sums of Certain Fourier Series	293
APPENDIX II Series Expansions of Elliptic Functions	295
APPENDIX III Table of Transformations	299
APPENDIX IV Bibliographies	315
 INDEX	 319