

Contents.

I. Introduction	1
I.1. The maser principle	1
I.2. The laser condition	2
I.3. Properties of laser light	5
a) Spatial coherence	5
b) Temporal coherence	6
c) Photon statistics	7
d) High intensity	7
e) Ultrashort pulses	7
I.4. Plan of the article	7
II. Optical resonators	9
II.1. Introduction	9
II.2. The Fabry-Perot resonator with plane parallel reflectors	11
a) Spatial distribution of modes	11
b) Diffraction losses	17
c) Three-dimensional resonator	18
II.3. Confocal resonator	19
a) Field outside the resonator	20
b) Field inside the resonator	21
c) Far field pattern of the confocal resonator	21
d) Phase shifts and losses	21
II.4. More general configurations	22
a) Confocal resonators with unequal square and rectangular apertures	22
b) Resonators with reflectors of unequal curvature	23
α) Large circular apertures	23
β) Large square aperture	23
II.5. Stability	23
III. Quantum mechanical equations of the light field and the atoms without losses	24
III.1. Quantization of the light field	24
III.2. Second quantization of the electron wave field	27
III.3. Interaction between radiation field and electron wave field	28
III.4. The interaction representation and the rotating wave approximation	29
III.5. The equations of motion in the Heisenberg picture	30
III.6. The formal equivalence of the system of atoms each having 2 levels with a system of $\frac{1}{2}$ spins	31
IV. Dissipation and fluctuation of quantum systems. The realistic laser equations	33
IV.1. Some remarks on homogeneous and inhomogeneous broadening	33
a) Natural linewidth	33
b) Inhomogeneous broadening	33
α) Impurity atoms in solids	33
β) Gases	34
γ) Semiconductors	34
c) Homogeneous broadening	34
α) Impurity atoms in solids	34
β) Gases	34
γ) Semiconductors	34

IV.2. A survey of IV.2.—IV.11	35
a) Definition of heatbaths (reservoirs)	35
b) The role of heatbaths	35
c) Classical Langevin and Fokker-Planck equations	36
α) Langevin equations	36
β) The Fokker-Planck equation	36
d) Quantum mechanical formulation: the total Hamiltonian	37
e) Quantum mechanical Langevin equations, Fokker-Planck equation and density matrix equation	38
α) Langevin equations	38
β) Density matrix equation	38
γ) Generalized Fokker-Planck equation	39
IV.3. Quantum mechanical Langevin equations: origin of quantum mechanical Langevin forces (the effect of heatbaths).	39
a) The field (one mode)	40
b) Electrons ("atoms")	42
IV.4. The requirement of quantum mechanical consistency	44
a) The field	44
b) Dissipation and fluctuations of the atoms	45
IV.5. The explicit form of the correlation functions of Langevin forces	46
a) The field	46
b) The N -level atom	46
IV.6. The complete laser equations	49
a) Quantum mechanically consistent equations for the operators b_k^\dagger and $(a_i^\dagger a_k)_\mu$	50
α) The field equations	50
β) The matter equations	50
b) Semiclassical equations	51
α) The field equations	51
β) The matter equations	51
IV.7. The density matrix equation	51
a) General derivation	51
b) Specialization of Eq. (IV.7.34)	56
α) Light mode	56
β) Atom	57
γ) The density matrix equation of the complete system of M laser modes and N atoms	58
IV.8. The evaluation of multi-time correlation functions by the single-time density matrix	59
IV.9. Generalized Fokker-Planck equation: definition of distribution functions	60
a) Field	61
α) Wigner distribution function and related representations	61
β) Transforms of the distribution functions: characteristic functions	63
γ) Calculation of expectation values by means of the distribution functions	64
b) Electrons	64
α) Distribution functions for a single electron	64
β) Characteristic functions	65
γ) Electrons and fields	65
IV.10. Equation for the laser distribution function (IV.9.22)	65
a) Comparison of the advantages of the Heisenberg and the Schrödinger representations	65
α) The Heisenberg representation	65
β) The Schrödinger representation	67
b) Final form of the generalized Fokker-Planck equation	70
IV.11. The calculation of multi-time correlation functions by means of the distribution function	71
V. Properties of quantized electromagnetic fields	73
V.1. Coherence properties of the classical and the quantized electro- magnetic field	73

a)	Classical description: definitions	73
α)	The complex analytical signal	73
β)	The average	74
γ)	The mutual coherence function	74
b)	Quantum theoretical coherence functions	76
α)	Elementary introductions	76
β)	Coherence functions	77
γ)	Coherent wave functions	78
δ)	Generation of coherent fields by classical sources (the forced harmonic oscillator)	80
V.2.	Uncertainty relations and limits of measurability	83
a)	Field and photon number	83
b)	Phase and photon number	85
α)	Heuristic considerations	85
β)	Exact treatment	85
c)	Field strength	87
V.3.	Spontaneous and stimulated emission and absorption	88
a)	Spontaneous emission	88
b)	Stimulated emission	90
c)	Comparison between spontaneous and stimulated emission rates	91
d)	Absorption	92
V.4.	Photon counting	93
a)	Quantum mechanical treatment, correlation functions	93
b)	Classical treatment of photon counting	94
V.5.	Coherence properties of spontaneous and stimulated emission. The spontaneous linewidth	97
VI.	Fully quantum mechanical solutions of the laser equations	99
VI.1.	Disposition	99
VI.2.	Summary of theoretical results and comparison with the experiments	101
a)	Qualitative discussion of the characteristic features of the laser output: homogeneously broadened line	102
b)	Quantitative results: single mode action	102
α)	The spectroscopic linewidth well above threshold	102
β)	The spectroscopic linewidth somewhat below threshold	103
γ)	The intensity (or amplitude) fluctuations	104
δ)	Photon statistics	107
VI.3.	The quantum mechanical Langevin equations for the solid state laser	112
a)	Field equations	113
b)	Matter equations	115
α)	The motion of the atomic dipole moment	115
1.	Dipole moment between levels j and k	115
2.	Dipole moment between levels j and $l \neq k$, j and between levels k and $l = j$, k	115
3.	Dipole moment between levels $i \neq k$, j and $l \neq k$, j	115
β)	The occupation numbers change	115
1.	For the laser levels j and k	115
2.	For the non-laser levels	116
VI.4.	Qualitative discussion of single mode operation	116
a)	The linear range (subthreshold region)	118
b)	The nonlinear range (at threshold and somewhat above)	119
α)	Phase diffusion	120
β)	Amplitude (intensity) fluctuations	120
c)	The nonlinear range at high inversion	120
d)	Exact elimination of all atomic coordinates	120
VI.5.	Quantitative treatment of a homogeneously broadened transition: emission below threshold (intensity, linewidth, amplification of signals)	120
a)	No external signals	120
α)	Single-mode linewidth below threshold	123
β)	Many modes below threshold	123
b)	External signals	124

VI.6. Exact elimination of atomic variables in the case of a homogeneously broadened line. Running or standing waves	125
α) Standing waves	125
β) Running waves	128
VI.7. Single mode operation above threshold, homogeneously broadened line	128
a) Lowest order	129
b) First order	130
c) Phase noise. Linewidth formula	130
d) Amplitude fluctuations	132
α) The special case of a moderate photon number	133
β) The special case of a big photon number	134
VI.8. Stability of amplitude. Spiking and damped oscillations. Single-mode operation, homogeneously broadened line	134
a) Qualitative discussion	135
b) Quantitative treatment	136
c) The special case $w_{13} \rightarrow \infty$ ("two level system")	137
VI.9. Qualitative discussion of two-mode operation	138
a) Some transformations	138
b) Both modes well below threshold	139
c) Modes somewhat above or somewhat below threshold	140
d) Both modes above threshold	141
α) $ \omega_1 - \omega_2 \gg 1/T$	142
β) $ \omega_1 - \omega_2 \lesssim 1/T$	143
VI.10. Gas laser and solid-state laser with an inhomogeneously broadened line. The van der Pol equation, single-mode operation	144
a) Solid-state laser with an inhomogeneously broadened line and an arbitrary number of levels	144
b) Gas laser	146
VI.11. Direct solution of the density matrix equation	146
VI.12. Reduction of the generalized Fokker-Planck equation for single-mode action	153
a) Expansion in powers of $N^{-\frac{1}{2}}$ (N : number of atoms)	154
b) Adiabatic elimination of the atomic variables	156
c) The Fokker-Planck equation	158
VI.13. Solution of the reduced Fokker-Planck equation	159
a) Steady state solution	159
b) Transient solution	166
VI.14. The Fokker-Planck equation for multimode action near threshold. Exact or nearly exact stationary solution	168
a) The explicit form of the Fokker-Planck equation	168
b) Theorem on the exact stationary solution of a Fokker-Planck equation	169
c) Nearly exact solution of (VI.14.1)	170
α) Normal multimode action	170
β) Phase locking of many modes	170
γ) A qualitative discussion of phase locking (example of three modes)	171
VI.15. The linear and quasi-linear solution of the general Fokker-Planck equation	172
a) Far below threshold	172
b) Well above threshold	172
VII. The semiclassical approach and its applications	173
VII.1. Spirit of the semiclassical approach. The equations for the solid state laser	173
a) The field equations	174
b) The material equations	175
c) Macroscopic treatment	178
α) Wave picture, inhomogeneous atomic line	178
β) Wave picture, homogeneous atomic line	178

γ) Wave picture, homogeneous atomic line, rotating wave approximation, slowly varying amplitude approximation . . .	179
δ) Mode picture, polarization waves	179
d) Extension to multilevel atoms	180
e) Systematics of the semiclassical approach	181
VII.2. Method of solution for the stationary state	182
a) Single-mode operation, general features	183
b) Two-mode operation, general features	184
α) Time-independent atomic response	185
β) Time-dependent atomic response	185
VII.3. The solid-state laser with a homogeneously broadened line. Single and multimode laser action	185
a) Single-mode operation	185
b) Multiple-mode operation	186
α) Equations for the photon densities of M modes	187
β) Equations for the frequency shift	187
VII.4. The solid-state laser with an inhomogeneously broadened Gaussian line. Single- and two-mode operation	187
a) One mode	187
α) Equation for the frequency shift	188
β) Equation for the photon density	189
b) Two modes	189
α) Equations for the photon densities \bar{n}_λ	189
β) Equations for the frequency shifts	189
c) Lorentzian line shape	190
VII.5. The solid-state laser with an inhomogeneously broadened line: multimode action	191
a) Normal multimode action	191
b) Combination tones	192
c) Frequency locking	193
VII.6. Equations of motion for the gas laser	194
VII.7. Single- and two-mode operation in gas lasers	197
a) Single-mode operation	197
α) Equation for the photon density	198
β) Equation for the frequency shift	199
b) Two-mode operation	199
α) Equations for the photon densities	200
β) Equations for the frequency shifts	201
VII.8. Some exactly solvable problems	201
a) Single-mode operation in solid state lasers	201
α) Homogeneously broadened line	202
1. Running waves	202
2. Standing waves in axial direction	202
β) Inhomogeneously broadened line, running waves	203
b) Single-mode in the gas laser	203
VII.9. External fields	203
a) The effect of a longitudinal magnetic field on the single spatial mode output	205
b) The field equations	206
c) The matter equations	208
d) Solution of the amplitude and frequency-determining Eqs. (VII.9.24), (VII.9.25)	210
VII.10. Ultrashort optical pulses: the principle of mode locking	213
a) Loss modulation by an externally driven modulator	215
b) Loss modulation by a saturable absorber	216
c) Gain modulation	216
d) Frequency modulation	217
e) Analogy to microwave circuits	217
VII.11. Ultrashort optical pulses: detailed treatment of loss modulation	217
a) Pulse shape and pulse width	222
b) Discussion of the results and of the range of validity	223
c) Numerical application	224

VII.12.	Super-radiance. Spin and photo echo	224
a)	Definition of super-radiant states	224
b)	Generation of super-radiant states	228
α)	Classical treatment of the spin motion	228
β)	Quantum theoretical treatment	229
c)	Classical description of super-radiant emission	231
d)	The spin-echo experiment	231
e)	The photo-echo experiment	232
f)	A further analogy between a spin $\frac{1}{2}$ system and a two-level system: the fictitious spin	234
VII.13.	Pulse propagation in laser-active media	236
a—c)	Steady state and self-pulsing	237
α)	The basic equations	237
β)	Stationary solution	238
γ)	Normalized amplitudes	238
δ)	Stability of the stationary solution	238
ϵ)	Transient build-up of the pulse	239
ζ)	Steady state pulse	241
η)	A simplified model	243
θ)	The special case $v = c$	244
d)	The π -pulse	245
e)	The 2π -pulse. (Self-induced transparency)	246
VII.14.	Derivation of rate equations	247
VIII.	Rate equations and their applications	249
VIII.1.	Formulation of rate equations and solution for the steady state (especially: threshold condition, pump power requirement, single versus multimode laser action)	249
a)	The rate equations	249
α)	The field equations	249
β)	The matter equations	250
b)	Treatment of the steady state	250
c)	The completely homogeneous case	251
α)	General formulation	251
β)	3-Level system, the lower transition is laser-active	252
γ)	Pump power at threshold	253
δ)	3-Level system, the upper transition is laser-active	253
ϵ)	4-Level system, laser action between the two middle levels	255
VIII.2.	The coexistence of modes on account of spatial inhomogeneities or an inhomogeneously broadened line	255
a)	Homogeneous line, but space-dependent modes (represented by standing waves)	255
α)	Axial modes with a different frequency distance from the line center	257
β)	Different losses	257
b)	Spatially inhomogeneous pumping, homogeneously broadened line	258
α)	Running waves	258
β)	Standing waves	258
c)	Inhomogeneously broadened line	259
VIII.3.	Laser cascades	259
a)	Matter equations	260
b)	Homogeneously broadened line and standing waves (modes in axial direction)	261
c)	Inhomogeneously broadened line and standing waves	261
d)	Discussion of an example	262
VIII.4.	Solution of the time-dependent rate equations. Relaxation oscillations.	264
a)	The 3-level system with laser action between the two lower levels	264
b)	3-Level system, laser action between the two upper levels	265
c)	4-Level system	266
d)	Approximate solution for small oscillations	266
VIII.5.	The giant pulse laser	267
a)	Semiquantitative treatment	268
b)	Quantitative treatment	269

IX. Further methods for dealing with quantum systems far from thermal equilibrium	271
IX.1. The general form of the density matrix equation	272
IX.2. Exact generalized Fokker-Planck equation: definition of the distribution function	274
IX.3. The exact generalized Fokker-Planck equation	275
IX.4. Derivation of the exact generalized Fokker-Planck equation	276
IX.5. Projection onto macroscopic variables	284
IX.6. Exact elimination of the atomic operators within quantum mechanical Langevin equations	286
IX.7. Rate equations in quantized form	287
IX.8. Exact elimination of the atomic operators from the density matrix equation	288
IX.9. Solution of the generalized field master Eq. (IX.8.12)	290
X. Appendix. Useful operator techniques	294
X.1. The harmonic oscillator	294
X.2. Operator relations for Bose operators	297
X.3. Formal solution of the Schrödinger equation	298
X.4. Disentangling theorem	299
X.5. Disentangling theorem for Bose operators	301
Sachverzeichnis (Deutsch-Englisch)	305
Subject Index (English-German)	313