Contents

Introduction 1

I. Basic Facts about Maxwell's Equations 25

1. Introduction 25
2. Maxwell's Differential Equations 25
3. Discontinuous Solutions and Discontinuity Conditions . . . 37
4. Time-Periodic Fields in Isotropic Media 51

II. The Electromagnetic Approach to Geometrical Optics in Isotropic Media 58

1. Introduction 58
2. The Field Vectors on the Wave Fronts 60
3. Properties of the Wave Fronts 64
4. Energy Flux and Rays in Isotropic Media 68
5. Fermat's Principle 72
6. Mechanical Interpretation of the Equations of the Rays . . 73

III. The Electromagnetic Approach to Geometrical Optics in Anisotropic Media 75

1. Introduction 75
2. The Field Vectors on the Wave Fronts 77
3. Duality of the Vector Triples 80
4. Fresnel's Differential Equation 82
5. Mathematical Analysis of the Fresnel Surface of Wave Normals . 83
6. Geometrical Analysis of the Fresnel Surface of Wave Normals . 88
7. Fresnel's Surface of Rays 98
8. Geometrical Relationship between the Two Fresnel Surfaces . 99
9. Wave Fronts in a Homogeneous Anisotropic Medium . . . 107
10. Rays in Anisotropic Media 109
11. Fermat's Principle in Anisotropic Media 112
12. Mechanical Interpretation of the Ray Paths 115

Appendix A 120

Appendix B 121

IV. Wave Fronts and Rays in Isotropic and Anisotropic Media . . 122

1. Introduction 122
2. Construction of Wave Fronts from Rays 124

3. Propagation of Wave Fronts across a Discontinuity in the Medium 131
4. Construction of Wave Fronts when the Initial Surface Is a Front: Huygens' Principle 136
5. Construction of Rays from Wave Fronts 142
6. Construction of Rays from Wave Fronts: Examples . . . 144

V. Propagation of the Geometrical Optics Field 147

1. Introduction 147
2. Propagation of Signals in Anisotropic Media 150
3. Propagation of Signals in Isotropic Media 158
4. Integration of the Transport Equations in Isotropic Media . . 167
5. Propagation of Signals across a Discontinuity in the Medium . 170
Appendix A. The Transport Equations in a Non-Euclidean Metric . 180
Appendix B. The Expansion Coefficient in Isotropic Media . . 184

VI. Pulse Solutions in Isotropic Media and Their Approximate Representation: The Regular Case 191

1. Introduction 191
2. An Integral Form of Maxwell's Equations 197
3. Derivation of the Discontinuity Conditions 201
4. Derivation of the Transport Equations 209
5. Solution of the Transport Equations 222
6. Initial Values for the Transport Equations 228
7. Summary 232

VII. Asymptotic Series Solution of Time-Harmonic Fields: The Dipole 237

1. Introduction 237
2. The Dipole Field in a Homogeneous Dielectric Medium . . 238
3. The Dipole Field in a Homogeneous Conducting Medium . . 245
4. The Dipole Field in a Non-homogeneous Medium . . . 246

VIII. Asymptotic Series Solution of Time-Harmonic Problems: The Regular Case 255

1. Introduction 255
2. The Duhamel Principle 257
3. The Asymptotic Series for Time-Harmonic Fields . . . 264
4. Determination of the Coefficients of the Asymptotic Series . . 270
5. Solution of the Transport Equations and Initial Values . . 276
6. Historical Remarks 279

IX. A Complex Integral Representation of Time-Harmonic Fields . 284

1. Introduction 284
2. The Stieltjes Transform of the Unit Pulse Solution . . . 285
3. Representation of the Amplitudes as Complex Integrals . . 295

X. Asymptotic Expansions of Time-Harmonic Fields in Fractional Powers 303

1. Introduction 303
2. Some Special Cases of the General Expansion Theorem . . 306
3. A General Theorem on Expansion in Fractional Powers . . 313
4. Proof of the Asymptotic Character of the Series 322

XI. The Electromagnetic Integrals of Diffraction Optics . . . 325

1. Introduction 325
2. Formulation of the Diffraction Problem 330
3. Construction of the Diffraction Integrals 342
4. A More Convenient Form of the Diffraction Integrals . . . 346
5. Example: The Reflection of a Plane Wave from a Spherical Mirror 350

XII. Asymptotic Evaluation of the Diffraction Integrals 361

1. Introduction 361
2. The Unit Pulse Solution of the Diffraction Problem . . . 365
3. Properties of the Phase Function 370
4. The Singularities of the Pulse Solution 375
5. Contributions to the Asymptotic Series from Interior Maxima and Minima of the Phase Function 381
6. Contributions to the Asymptotic Series from Boundary Maxima and Minima of the Phase Function 388
7. Contributions to the Asymptotic Series from Saddle Points of the Phase Function 396
8. The Behavior of the Diffraction Integrals at Infinity . . . 403
9. The Geometrical Significance of the Pulse Solution . . . 406

XIII. The Optical Diffraction Integral for Unpolarized Light . . 410

1. Introduction 410
2. Representation of the Vector Diffraction Integrals by Two Scalar Functions 414
3. The Characterization of Unpolarized Light 417
4. Average Energy Density and Average Flux Density . . . 419
5. The Scalar Diffraction Integral for Unpolarized Light . . . 425
6. Basic Facts about the Diffraction Patterns of Unpolarized Waves 426
7. The Diffraction Pattern of a Spherical Wave 430
8. Asymptotic Expansion of the Diffraction Pattern of a Perfectly Spherical Wave 435

XIV. Some Applications of the Asymptotic Series 457

1. Introduction 457
2. Reflection of a Plane Wave from a Perfectly Conducting Paraboloid of Revolution 459

3. Diffraction of a Plane Wave Incident on a Perfectly Conducting Half-Plane 469
4. The Propagation of a Wave in an Inhomogeneous Medium . . 482
5. Geometrical Diffraction Theory 494
6. Applications to Other Branches of Mathematical Physics . . 499

Appendix. Vector Analysis Formulas 505

Selected References 519

Author Index 523

Subject Index 526