

CONTENTS

	PAGE
PREFACE TO THE FIRST EDITION	v
PREFACE TO THE SECOND EDITION	ix
HISTORICAL INTRODUCTION	xxi
I. BASIC PROPERTIES OF THE ELECTROMAGNETIC FIELD	1
1.1. The Electromagnetic Field	1
1.1.1. Maxwell's equations	1
1.1.2. Material equations	2
1.1.3. Boundary conditions at a surface of discontinuity	4
1.1.4. The energy law of the electromagnetic field	7
1.2. The Wave Equation and the Velocity of Light	10
1.3. Scalar Waves	14
1.3.1. Plane waves	14
1.3.2. Spherical waves	15
1.3.3. Harmonic waves. The phase velocity	16
1.3.4. Wave packets. The group velocity	18
1.4. Vector Waves	23
1.4.1. The general electromagnetic plane wave	23
1.4.2. The harmonic electromagnetic plane wave	24
(a) Elliptic polarization	25
(b) Linear and circular polarization	28
(c) Characterization of the state of polarization by Stokes parameters	30
1.4.3. Harmonic vector waves of arbitrary form	32
1.5. Reflection and Refraction of a Plane Wave	36
1.5.1. The laws of reflection and refraction	36
1.5.2. Fresnel formulae	38
1.5.3. The reflectivity and transmissivity; polarization on reflection and refraction	41
1.5.4. Total reflection	47
1.6. Wave Propagation in a Stratified Medium. Theory of Dielectric Films	51
1.6.1. The basic differential equations	52
1.6.2. The characteristic matrix of a stratified medium	55
(a) A homogeneous dielectric film	57
(b) A stratified medium as a pile of thin homogeneous films	58
1.6.3. The reflection and transmission coefficients	59
1.6.4. A homogeneous dielectric film	61
1.6.5. Periodically stratified media	66
II. ELECTROMAGNETIC POTENTIALS AND POLARIZATION	71
2.1. The Electrodynamic Potentials in the Vacuum	72
2.1.1. The vector and scalar potentials	72
2.1.2. Retarded potentials	74

	PAGE
2.2. Polarization and Magnetization	76
2.2.1. The potentials in terms of polarization and magnetization	76
2.2.2. Hertz vectors	79
2.2.3. The field of a linear electric dipole	81
2.3. The Lorentz–Lorenz Formula and Elementary Dispersion Theory	84
2.3.1. The dielectric and magnetic susceptibilities	84
2.3.2. The effective field	85
2.3.3. The mean polarizability: the Lorentz–Lorenz formula	87
2.3.4. Elementary theory of dispersion	90
2.4. Propagation of Electromagnetic Waves Treated by Integral Equations	98
2.4.1. The basic integral equation	99
2.4.2. The Ewald–Oseen extinction theorem and a rigorous derivation of the Lorentz–Lorenz formula	100
2.4.3. Refraction and reflection of a plane wave, treated with the help of the Ewald–Oseen extinction theorem	104
 III. FOUNDATIONS OF GEOMETRICAL OPTICS	 109
3.1. Approximation for Very Short Wavelengths	109
3.1.1. Derivation of the eikonal equation	110
3.1.2. The light rays and the intensity law of geometrical optics	113
3.1.3. Propagation of the amplitude vectors	117
3.1.4. Generalizations and the limits of validity of geometrical optics	119
3.2. General Properties of Rays	121
3.2.1. The differential equation of light rays	121
3.2.2. The laws of refraction and reflection	124
3.2.3. Ray congruences and their focal properties	126
3.3. Other Basic Theorems of Geometrical Optics	127
3.3.1. Lagrange’s integral invariant	127
3.3.2. The principle of Fermat	128
3.3.3. The theorem of Malus and Dupin and some related theorems	130
 IV. GEOMETRICAL THEORY OF OPTICAL IMAGING	 133
4.1. The Characteristic Functions of Hamilton	133
4.1.1. The point characteristic	133
4.1.2. The mixed characteristic	135
4.1.3. The angle characteristic	137
4.1.4. Approximate form of the angle characteristic of a refracting surface of revolution	138
4.1.5. Approximate form of the angle characteristic of a reflecting surface of revolution	141
4.2. Perfect Imaging	143
4.2.1. General theorems	143
4.2.2. Maxwell’s “fish-eye”	147
4.2.3. Stigmatic imaging of surfaces	149

	PAGE
4.3. Projective Transformation (Collineation) with Axial Symmetry	150
4.3.1. General formulae	151
4.3.2. The telescopic case	154
4.3.3. Classification of projective transformations	154
4.3.4. Combination of projective transformations	155
4.4. Gaussian Optics	157
4.4.1. Refracting surface of revolution	157
4.4.2. Reflecting surface of revolution	160
4.4.3. The thick lens	161
4.4.4. The thin lens	163
4.4.5. The general centred system	164
4.5. Stigmatic Imaging with Wide-angle Pencils	166
4.5.1. The sine condition	167
4.5.2. The Herschel condition	169
4.6. Astigmatic Pencils of Rays	169
4.6.1. Focal properties of a thin pencil	169
4.6.2. Refraction of a thin pencil	171
4.7. Chromatic Aberration. Dispersion by a Prism	174
4.7.1. Chromatic aberration	174
4.7.2. Dispersion by a prism	177
4.8. Photometry and Apertures	181
4.8.1. Basic concepts of photometry	181
4.8.2. Stops and pupils	186
4.8.3. Brightness and illumination of images	188
4.9. Ray Tracing	190
4.9.1. Oblique meridional rays	191
4.9.2. Paraxial rays	193
4.9.3. Skew rays	194
4.10. Design of Aspheric Surfaces	197
4.10.1. Attainment of axial stigmatism	197
4.10.2. Attainment of aplanatism	200
V. GEOMETRICAL THEORY OF ABERRATIONS	203
5.1. Wave and Ray Aberrations; the Aberration Function	203
5.2. The Perturbation Eikonal of Schwarzschild	207
5.3. The Primary (Seidel) Aberrations	211
5.4. Addition Theorem for the Primary Aberrations	218
5.5. The Primary Aberration Coefficients of a General Centred Lens System	220
5.5.1. The Seidel formulae in terms of two paraxial rays	220
5.5.2. The Seidel formulae in terms of one paraxial ray	224
5.5.3. Petzval's theorem	225
5.6. Example: The Primary Aberrations of a Thin Lens	226
5.7. The Chromatic Aberration of a General Centred Lens System	230

	PAGE
VI. IMAGE-FORMING INSTRUMENTS	233
6.1. The Eye	233
6.2. The Camera	235
6.3. The Refracting Telescope	239
6.4. The Reflecting Telescope	245
6.5. Instruments of Illumination	250
→ 6.6. The Microscope	251
VII. ELEMENTS OF THE THEORY OF INTERFERENCE AND INTERFEROMETERS	256
7.1. Introduction	257
7.2. Interference of Two Monochromatic Waves	257
7.3. Two-beam Interference: Division of Wave-front	260
7.3.1. Young's experiment	260
7.3.2. Fresnel's mirrors and similar arrangements	261
7.3.3. Fringes with quasi-monochromatic and white light	264
7.3.4. Use of slit sources; visibility of fringes	265
7.3.5. Application to the measurement of optical path difference: the Rayleigh interferometer	268
7.3.6. Application to the measurement of angular dimensions of sources: the Michelson stellar interferometer	271
7.4. Standing Waves	277
7.5. Two-beam Interference: Division of Amplitude	281
7.5.1. Fringes with a plane parallel plate	281
7.5.2. Fringes with thin films; the Fizeau interferometer	286
7.5.3. Localization of fringes	291
7.5.4. The Michelson interferometer	300
7.5.5. The Twyman-Green and related interferometers	302
→ 7.5.6. Fringes with two identical plates: the Jamin interferometer and interference microscopes	306
7.5.7. The Mach-Zehnder interferometer; the Bates wave-front shearing interferometer	312
7.5.8. The coherence length; the application of two-beam interference to the study of the fine structure of spectral lines	316
7.6. Multiple-beam Interference	323
7.6.1. Multiple-beam fringes with a plane parallel plate	323
7.6.2. The Fabry-Perot interferometer	329
7.6.3. The application of the Fabry-Perot interferometer to the study of the fine structure of spectral lines	333
7.6.4. The application of the Fabry-Perot interferometer to the comparison of wavelengths	338
7.6.5. The Lummer-Gehrcke interferometer	341
7.6.6. Interference filters	347
7.6.7. Multiple-beam fringes with thin films	351

	PAGE
7.6.8. Multiple-beam fringes with two plane parallel plates	360
(a) Fringes with monochromatic and quasi-monochromatic light	360
(b) Fringes of superposition	364
7.7. The Comparison of Wavelengths with the Standard Metre	367
VIII. ELEMENTS OF THE THEORY OF DIFFRACTION	370
8.1. Introduction	370
8.2. The Huygens–Fresnel Principle	370
8.3. Kirchhoff's Diffraction Theory	375
8.3.1. The integral theorem of Kirchhoff	375
8.3.2. Kirchhoff's diffraction theory	378
8.3.3. Fraunhofer and Fresnel diffraction	382
8.4. Transition to a Scalar Theory	387
8.4.1. The image field due to a monochromatic oscillator	387
8.4.2. The total image field	390
8.5. Fraunhofer Diffraction at Apertures of Various Forms	392
8.5.1. The rectangular aperture and the slit	393
8.5.2. The circular aperture	395
8.5.3. Other forms of aperture	398
8.6. Fraunhofer Diffraction in Optical Instruments	401
8.6.1. Diffraction gratings	401
(a) The principle of the diffraction grating	401
(b) Types of grating	407
(c) Grating spectrographs	412
8.6.2. Resolving power of image-forming systems	414
8.6.3. Image formation in the microscope	418
(a) Incoherent illumination	418
(b) Coherent illumination—Abbe's theory	419
(c) Coherent illumination—Zernike's phase contrast method of observation	424
8.7. Fresnel Diffraction at a Straight Edge	428
8.7.1. The diffraction integral	428
8.7.2. Fresnel's integrals	430
8.7.3. Fresnel diffraction at a straight edge	433
8.8. The Three-dimensional Light Distribution near Focus	435
8.8.1. Evaluation of the diffraction integral in terms of Lommel functions	435
8.8.2. The distribution of intensity	439
(a) Intensity in the geometrical focal plane	441
(b) Intensity along the axis	441
(c) Intensity along the boundary of the geometrical shadow	441
8.8.3. The integrated intensity	442
8.8.4. The phase behaviour	445
8.9. The Boundary Diffraction Wave	449

	PAGE
8.10. Gabor's Method of Imaging by Reconstructed Wave-fronts	453
8.10.1. Producing the positive hologram	453
8.10.2. The reconstruction	455
IX. THE DIFFRACTION THEORY OF ABERRATIONS	459
9.1. The Diffraction Integral in the Presence of Aberrations	460
9.1.1. The diffraction integral	462
9.1.2. The displacement theorem. Change of reference sphere	462
9.1.3. A relation between the intensity and the average deformation of wave-fronts	463
9.2. Expansion of the Aberration Function	464
9.2.1. The circle polynomials of Zernike	464
9.2.2. Expansion of the aberration function	466
9.3. Tolerance Conditions for Primary Aberrations	468
9.4. The Diffraction Pattern Associated with a Single Aberration	473
9.4.1. Primary spherical aberration	475
9.4.2. Primary coma	477
9.4.3. Primary astigmatism	479
9.5. Imaging of Extended Objects	480
9.5.1. Coherent illumination	481
9.5.2. Incoherent illumination	484
X. INTERFERENCE AND DIFFRACTION WITH PARTIALLY COHERENT LIGHT	491
10.1. Introduction	491
10.2. A Complex Representation of Real Polychromatic Fields	494
10.3. The Correlation Functions of Light Beams	499
10.3.1. Interference of two partially coherent beams. The mutual coherence function and the complex degree of coherence	499
10.3.2. Spectral representation of mutual coherence	503
10.4. Interference and Diffraction with Quasi-monochromatic Light	505
10.4.1. Interference with quasi-monochromatic light. The mutual intensity	505
10.4.2. Calculation of mutual intensity and degree of coherence for light from an extended incoherent quasi-monochromatic source	508
(a) The Van Cittert-Zernike theorem	508
(b) Hopkins' formula	512
10.4.3. An example	513
10.4.4. Propagation of mutual intensity	516

	PAGE
10.5. Some Applications	518
10.5.1. The degree of coherence in the image of an extended incoherent quasi-monochromatic source	518
10.5.2. The influence of the condenser on resolution in a microscope	522
(a) Critical illumination	522
(b) Köhler's illumination	524
10.5.3. Imaging with partially coherent quasi-monochromatic illumination	526
(a) Transmission of mutual intensity through an optical system	526
(b) Images of transilluminated objects	528
10.6. Some Theorems Relating to Mutual Coherence	532
10.6.1. Calculation of mutual coherence for light from an incoherent source	532
10.6.2. Propagation of mutual coherence	534
10.7. Rigorous Theory of Partial Coherence	535
10.7.1. Wave equations for mutual coherence	535
10.7.2. Rigorous formulation of the propagation law for mutual coherence	537
10.7.3. The coherence time and the effective spectral width	540
10.8. Polarization Properties of Quasi-monochromatic Light	544
10.8.1. The coherency matrix of a quasi-monochromatic plane wave	544
(a) Completely unpolarized light (Natural light)	548
(b) Completely polarized light	549
10.8.2. Some equivalent representations. The degree of polarization of a light wave	550
10.8.3. The Stokes parameters of a quasi-monochromatic plane wave	554
XI. RIGOROUS DIFFRACTION THEORY	556
11.1. Introduction	556
11.2. Boundary Conditions and Surface Currents	557
11.3. Diffraction by a Plane Screen: Electromagnetic Form of Babinet's Principle	559
11.4. Two-dimensional Diffraction by a Plane Screen	560
11.4.1. The scalar nature of two-dimensional electromagnetic fields	560
11.4.2. An angular spectrum of plane waves	561
11.4.3. Formulation in terms of dual integral equations	564
11.5. Two-dimensional Diffraction of a Plane Wave by a Half-plane	565
11.5.1. Solution of the dual integral equations for E -polarization	565
11.5.2. Expression of the solution in terms of Fresnel integrals	567
11.5.3. The nature of the solution	570
11.5.4. The solution for H -polarization	574
11.5.5. Some numerical calculations	575
11.5.6. Comparison with approximate theory and with experimental results	577

	PAGE
11.6. Three-dimensional Diffraction of a Plane Wave by a Half-plane	578
11.7. Diffraction of a Localized Source by a Half-plane	580
11.7.1. A line-current parallel to the diffracting edge	580
11.7.2. A dipole	584
11.8. Other Problems	587
11.8.1. Two parallel half-planes	587
11.8.2. An infinite stack of parallel, staggered half-planes	589
11.8.3. A strip	589
11.8.4. Further problems	591
11.9. Uniqueness of Solution	591
XII. DIFFRACTION OF LIGHT BY ULTRASONIC WAVES	593
12.1. Qualitative Description of the Phenomenon and Summary of Theories Based on Maxwell's Differential Equations	593
12.1.1. Qualitative description of the phenomenon	593
12.1.2. Summary of theories based on Maxwell's equations	596
12.2. Diffraction of Light by Ultrasonic Waves as Treated by the Integral Equation Method	599
12.2.1. Integral equation for E -polarization	600
12.2.2. The trial solution of the integral equation	601
12.2.3. Expressions for the amplitudes of the light waves in the diffracted and reflected spectra	603
12.2.4. Solution of the equations by a method of successive approximations	604
12.2.5. Expressions for the intensities of the first and second order lines for some special cases	607
12.2.6. Some qualitative results	608
12.2.7. The Raman-Nath approximation	609
XIII. OPTICS OF METALS	611
13.1. Wave Propagation in a Conductor	611
13.2. Refraction and Reflection at a Metal Surface	615
13.3. Elementary Electron Theory of the Optical Constants of Metals	624
13.4. Wave Propagation in a Stratified Conducting Medium. Theory of Metallic Films	627
13.4.1. An absorbing film on a transparent substrate	627
13.4.2. A transparent film on an absorbing substrate	632
13.5. Diffraction by a Conducting Sphere; Theory of Mie	633
13.5.1. Mathematical solution of the problem	634
(a) Representation of the field in terms of Debye's potentials	634
(b) Series expansions for the field components	639
(c) Summary of formulae relating to the spherical harmonics and to the cylindrical functions	645

CONTENTS

xix

	PAGE
13.5.2. Some consequences of Mie's formulae	647
(a) The partial waves	647
(b) Limiting cases	649
(c) Intensity and polarization of the scattered light	652
13.5.3. Total scattering and extinction	656
(a) Some general considerations	656
(b) Computational results	661
XIV. OPTICS OF CRYSTALS	665
14.1. The Dielectric Tensor of an Anisotropic Medium	665
14.2. The Structure of a Monochromatic Plane Wave in an Anisotropic Medium	667
14.2.1. The phase velocity and the ray velocity	667
14.2.2. Fresnel's formulae for the propagation of light in crystals	670
14.2.3. Geometrical constructions for determining the velocities of propagation and the directions of vibration	673
(a) The ellipsoid of wave normals	673
(b) The ray ellipsoid	676
(c) The normal surface and the ray surface	676
14.3. Optical Properties of Uniaxial and Biaxial Crystals	678
14.3.1. The optical classification of crystals	678
14.3.2. Light propagation in uniaxial crystals	679
14.3.3. Light propagation in biaxial crystals	681
14.3.4. Refraction in crystals	684
(a) Double refraction	684
(b) Conical refraction	686
14.4. Measurements in Crystal Optics	690
14.4.1. The Nicol prism	690
14.4.2. Compensators	691
(a) The quarter-wave plate	691
(b) Babinet's compensator	692
(c) Soleil's compensator	694
(d) Berek's compensator	694
14.4.3. Interference with crystal plates	694
14.4.4. Interference figures from uniaxial crystal plates	698
14.4.5. Interference figures from biaxial crystal plates	701
14.4.6. Location of optic axes and determination of the principal refractive indices of a crystalline medium	702
14.5. Stress Birefringence and Form Birefringence	703
14.5.1. Stress birefringence	703
14.5.2. Form birefringence	705
14.6. Absorbing Crystals	708
14.6.1. Light propagation in an absorbing anisotropic medium	708
14.6.2. Interference figures from absorbing crystal plates	713
(a) Uniaxial crystals	714
(b) Biaxial crystals	715
14.6.3. Dichroic polarizers	716

	PAGE
APPENDICES	719
I. <i>The Calculus of Variations</i>	719
1. Euler's equations as necessary conditions for an extremum	719
2. Hilbert's independence integral and the Hamilton–Jacobi equation	720
3. The field of extremals	722
4. Determination of all extremals from the solution of the Hamilton–Jacobi equation	724
5. Hamilton's canonical equations	725
6. The special case when the independent variable does not appear explicitly in the integrand	726
7. Discontinuities	727
8. Weierstrass' and Legendre's conditions (sufficiency conditions for an extremum)	729
9. Minimum of the variational integral when one end point is constrained to a surface	731
10. Jacobi's criterion for a minimum	732
11. Example I: Optics	732
12. Example II: Mechanics of material points	734
II. <i>Light Optics, Electron Optics and Wave Mechanics</i>	738
1. The Hamiltonian analogy in elementary form	738
2. The Hamiltonian analogy in variational form	740
3. Wave mechanics of free electrons	743
4. The application of optical principles to electron optics	745
III. <i>Asymptotic Approximations to Integrals</i>	747
1. The method of steepest descent	747
2. The method of stationary phase	752
3. Double integrals	753
IV. <i>The Dirac Delta Function</i>	755
V. <i>A Mathematical Lemma used in the Rigorous Derivation of the Lorentz–Lorenz Law (§ 2.4.2)</i>	760
VI. <i>Propagation of Discontinuities in an Electromagnetic Field (§ 3.1.1)</i>	763
1. Relations connecting discontinuous changes in field vectors	763
2. The field on a moving discontinuity surface	765
VII. <i>The Circle Polynomials of Zernike (§ 9.2.1)</i>	767
1. Some general considerations	767
2. Explicit expressions for the radial polynomials $R_n^{\pm m}(\rho)$	769
VIII. <i>Proof of an Inequality (§ 10.7.3)</i>	773
IX. <i>Evaluation of Two Integrals (§ 12.2.2)</i>	775
AUTHOR INDEX	779
SUBJECT INDEX	790