CONTENTS

CHAPTER I. THE FOUNDATIONS OF THE SPECIAL THEORY OF RELATIVITY. HISTORICAL SURVEY 1

§ 1. The relativity principle of mechanics. The Galilean transformation 1

§ 2. The special principle of relativity 4

§ 3. Invariance of the phase of a plane wave 6

§ 4. Transformation of the characteristics of a plane wave. . . 8

§ 5. The Doppler effect 8

§ 6. The velocity of light *in vacuo* 10

§ 7. The velocity of light in refractive media 15

§ 8. Hoek's and Fizeau's experiments. 17

§ 9. Lorentz's theory of electrons 20

§ 10. Agreement between the ether theory and the principle of relativity as regards all effects of the first order. Fermat's principle . 22

§ 11. The aberration of light 25

§ 12. Michelson's experiment 26

§ 13. The contraction hypothesis 28

§ 14. Validity of the principle of relativity for all physical phenomena . 29

CHAPTER II. RELATIVISTIC KINEMATICS 31

§ 15. Simultaneity of events 31

§ 16. The relativity of simultaneity 33

§ 17. The special Lorentz transformation 36

§ 18. The most general Lorentz transformation 41

§ 19. Contraction of bodies in motion 44

§ 20. The retardation of moving clocks. The clock paradox . . 48

§ 21. Transformation of particle velocities 51

§ 22. Successive Lorentz transformations. The Thomas precession . 53

§ 23. Transformation of the characteristics of a wave according to the theory of relativity 56

§ 24. The ray velocity in moving bodies 58

§ 25. The Doppler effect, the aberration of light, and the dragging phenomenon according to the theory of relativity 62

CHAPTER III. RELATIVISTIC MECHANICS 67

§ 26. Momentum and mass of a particle 67

§ 27. Force, work, kinetic energy 70

§ 28. Transformation equations for momentum energy and force . 71

§ 29. Hyperbolic motion. Motion of an electrically charged particle in a constant magnetic field 74

§ 30. Equivalence of energy and mass 77

§ 31. Inelastic collisions. Mass of a closed system of particles . . 82
§ 32. Experimental verification of relativistic mechanics . . 85
CHAPTER IV. FOUR-DIMENSIONAL FORMULATION OF THE THEORY OF RELATIVITY: TENSOR CALCULUS . . 92
§ 33. Four-dimensional representation of the Lorentz transformation . 92
§ 34. Lorentz contraction and retardation of moving clocks in four-dimensional representation 96
§ 35. Covariance of the laws of nature in four-dimensional formulation . 97
§ 36. The four-dimensional line element or interval. Four-vectors . 99
§ 37. Four-velocity and acceleration. Wave-number vector. Four-ray velocity 101
§ 38. Four-momentum. Four-force. Fundamental equations of point mechanics in four-dimensional vector form . . . 104
§ 39. Tensors of rank 2 108
§ 40. Angular momentum and moment of force in four-dimensional representation 110
§ 41. Tensors of arbitrary rank 111
§ 42. Pseudo-tensors 112
§ 43. The Levi-Civita symbol 113
§ 44. Dual tensors 114
§ 45. Infinitesimal Lorentz transformations. Lorentz transformations without rotation 117
§ 46. Successive Lorentz transformations 118
§ 47. Successive rest systems of a particle in arbitrary rectilinear motion and in constant circular motion 121
§ 48. Tensor and pseudo-tensor fields. Tensor analysis . . 125
§ 49. Gauss's theorem in four-dimensional space . . . 128
§ 50. The fundamental equations of mechanics for incoherent matter . 130
§ 51. The kinetic energy-momentum tensor 136
CHAPTER V. ELECTRODYNAMICS IN THE VACUUM . . 139
§ 52. The fundamental equations of electrodynamics in the vacuum. Four-current density for electric charge . . . 139
§ 53. Covariance of the fundamental equations of electrodynamics under Lorentz transformations. The electromagnetic field tensor 141
§ 54. The four-potential. Gauge transformation . . . 143
§ 55. Four-dimensional integral representation of the four-potential . 144
§ 56. Retarded potentials. Liénard-Wiechert's potentials for point charges 147
§ 57. The field of a uniformly moving point charge . . . 151
§ 58. The electromagnetic forces acting on charged matter . . 154
§ 59. Variational principle of electrodynamics 157
§ 60. The electromagnetic energy-momentum tensor . . . 159
§ 61. The total energy-momentum tensor 161

CHAPTER VI. GENERAL CLOSED SYSTEMS. MECHANICS OF ELASTIC CONTINUA. FIELD THEORY 163
§ 62. Definition of a closed system 163
§ 63. Four-momentum and angular momentum four-tensor for a closed system 166
§ 64. Centre of mass 170
§ 65. The fundamental equations of mechanics in elastic continua . 173
§ 66. Transformation of elastic stress, momentum density, and energy density 179
§ 67. Perfect fluids 181
§ 68. Scalar meson fields. General field theory 184

CHAPTER VII. NON-CLOSED SYSTEMS. ELECTRODYNAMICS IN DIELECTRIC AND PARAMAGNETIC SUBSTANCES. THERMODYNAMICS 188
§ 69. General properties of non-closed systems 188
§ 70. Static non-closed systems 191
§ 71. Electrostatic systems. Classical models of the electron . . 192
§ 72. The fundamental equations of electrodynamics in stationary matter. 195
§ 73. Minkowski's field equations in uniformly moving bodies . . 196
§ 74. The constitutive equations in four-dimensional language. Boundary conditions 201
§ 75. Electromagnetic energy-momentum tensor and four-force density 202
§ 76. The propagation velocity of the energy of a light wave in a moving refractive body 206
§ 77. The laws of thermodynamics in stationary matter . . 211
§ 78. Transformation properties of the thermodynamical quantities . 212
§ 79. Four-dimensional formulation of the laws of thermodynamics . 214
§ 80. Ideal monatomic gases 215
§ 81. Black-body radiation 216

CHAPTER VIII. THE FOUNDATIONS OF THE GENERAL THEORY OF RELATIVITY 218
§ 82. The general principle of relativity 218
§ 83. The principle of equivalence 220
§ 84. Uniformly rotating systems of coordinates. Space and time in the general theory of relativity 222
§ 85. Non-Euclidean geometry. The metric tensor . . . 226
§ 86. Geodesic lines 228
§ 87. Determination of the metric tensor by direct measurements. Geometry in n-dimensional space 231
§ 88. General accelerated systems of reference. The most general admissible space-time transformations 233

§ 89. Space and time measurements in an arbitrary system of reference. Experimental determination of the functions g_{ik} . . 237
§ 90. The spatial geometry in the rotating system of reference . 240
§ 91. The time tracks of free particles and light rays . . . 244
§ 92. The dynamical gravitational potentials 245
§ 93. The rate of a moving standard clock in a gravitational field . 247
§ 94. Transformation of coordinates inside a fixed system of reference . 248
§ 95. Further simple examples of accelerated systems of reference . 250
§ 96. Rigid systems of reference with an arbitrary motion of the origin 253
§ 97. Rigid frames of reference moving in the direction of the X-axis . 255
§ 98. The clock paradox 258

CHAPTER IX. PERMANENT GRAVITATIONAL FIELDS. TENSOR CALCULUS IN A GENERAL RIEMANNIAN SPACE . . 264
§ 99. Four-dimensional formulation of the general principle of relativity and of the principle of equivalence 264
§ 100. Contravariant and covariant components of a four-vector . 266
§ 101. Tensor algebra 269
§ 102. Pseudo-tensors. Dual tensors 270
§ 103. Geodesic lines. Christoffel's formulae 272
§ 104. Local systems of inertia 274
§ 105. Parallel displacement of vectors 276
§ 106. Tensor analysis. Covariant differentiation . . . 279
§ 107. The curvature tensor 284
§ 108. The contracted forms of the curvature tensor . . . 286

CHAPTER X. THE INFLUENCE OF GRAVITATIONAL FIELDS ON PHYSICAL PHENOMENA 288
§ 109. Mechanics of free particles in the presence of gravitational fields 288
§ 110. Momentum and mass of a particle. Gravitational force . . 290
§ 111. Total energy of a particle in a stationary gravitational field . 294
§ 112. General point mechanics 295
§ 113. Time-orthogonal systems of coordinates. Elimination of the dynamical potentials 296
§ 114. Mechanics of continuous systems 298
§ 115. The electromagnetic field equations 302
§ 116. Electromagnetic force and energy-momentum tensor . . 305
§ 117. Propagation of light in a static gravitational field. Fermat's principle 308

CHAPTER XI. THE FUNDAMENTAL LAWS OF GRAVITATION IN THE GENERAL THEORY OF RELATIVITY . . . 310
§ 118. The gravitational field equations 310
§ 119. The linear approximation for weak fields . . . 313

§ 120. Simple applications of the linear equations for weak fields. The relativity of centrifugal forces and Coriolis forces . . 315
§ 121. Equivalent systems of coordinates. Systems with spherical symmetry 321
§ 122. Static systems with spherical symmetry 323
§ 123. Schwarzschild's exterior solution 325
§ 124. Schwarzschild's solution for the interior of a perfect fluid . 328
§ 125. The variational principle for gravitational fields . . . 333
§ 126. The laws of conservation of energy and momentum . . 337
§ 127. Different expressions for the densities of energy and momentum 341
§ 128. The gravitational mass and total energy and momentum of an isolated system 342

CHAPTER XII. EXPERIMENTAL VERIFICATION OF THE GENERAL THEORY OF RELATIVITY. COSMOLOGICAL PROBLEMS . 346
§ 129. The gravitational shift of spectral lines 346
§ 130. The advance of the perihelion of Mercury . . . 348
§ 131. The gravitational deflexion of light 353
§ 132. Cosmological models 356
§ 133. The Einstein universe 357
§ 134. The de Sitter universe 362

APPENDIXES 371
1. Gauss's theorem 371
2. The transformation equations for the four-current density . . 372
3. Plane waves in a homogeneous isotropic substance . . . 373
4. Transformation of the gravitational field variables $\gamma_{\iota\kappa}$, γ_ι, χ, $\omega_{\iota\kappa}$ by a change of coordinates inside a definite system of reference . 374
5. Dual tensors in a three-dimensional space 375
6. The condition for flat space 376
7. The action principle and the Hamiltonian equations for a particle in an arbitrary gravitational field 378
8. The connexion between the determinants of the space-time metric tensor and the spatial metric tensor 381
9. The derivatives of the function $\mathfrak{L}$ with respect to g^{lm}_k and g^{lm} and some identities containing these derivatives 382

AUTHOR INDEX 384

SUBJECT INDEX 385