CONTENTS

Preface ix

Chapter 1 The Physical Basis of Quantum Mechanics 1

1 Experimental Background 2
Inadequacy of classical physics. Summary of principal experiments and inferences.

2 The Old Quantum Theory 4
Bohr-Sommerfeld quantization rules. Practical difficulties. Conceptual difficulties. Quantum-mechanical viewpoint.

3 Uncertainty and Complementarity 7
Uncertainty principle. Complementarity principle. Limitations on experiment.

4 Discussion of Measurement 9
Localization experiment. Momentum determination experiment. Analysis of diffraction experiment. Discussion of diffraction experiment.

5 Wave Packets in Space and Time 14
Space packets. Time packets. Wave formalism.

Chapter 2 The Schrödinger Wave Equation 19

6 Development of the Wave Equation 20
Traveling harmonic waves. Need for a wave equation. The one-

dimensional wave equation. Extension to three dimensions. Inclusion of forces.

7 Interpretation of the Wave Function 24
Statistical interpretation. Normalization of ψ. Probability current density. Expectation value. Ehrenfest's theorem.

8 Energy Eigenfunctions 30
Separation of the wave equation. Significance of the separation constant E. Boundary conditions at great distances. Continuity conditions. Boundary conditions for infinite potential energy. Energy eigenvalues in one dimension. Discrete energy levels. Continuous energy eigenvalues. Discrete and continuous eigenvalues in three dimensions.

9 One-dimensional Square Well Potential 37
Perfectly rigid walls. Finite potential step. Energy levels. Parity. A simplified solution.

Chapter 3 Eigenfunctions and Eigenvalues 45

10 Interpretative Postulates and Energy Eigenfunctions 46
Dynamical variables as operators. Expansion in eigenfunctions. The total-energy operator. Normalization in a box. Orthonormality of energy eigenfunctions. Reality of the energy eigenvalues. Expansion in energy eigenfunctions. The closure property. Probability function and expectation value. General solution of the Schrödinger equation.

11 Momentum Eigenfunctions 53
Form of the eigenfunctions. Box normalization. The Dirac δ function. A representation of the δ function. Normalization in terms of the δ function. Some properties of the δ function. Closure. Expansion in momentum eigenfunctions. Probability function and expectation value.

12 Motion of a Free Wave Packet in One Dimension 60
The minimum uncertainty product. Form of the minimum packet. Momentum expansion coefficients. Change with time of a minimum packet. Classical limit.

Chapter 4 Discrete Eigenvalues: Bound States 66

13 Linear Harmonic Oscillator 66
Asymptotic behavior. Energy levels. Zero-point energy. Hermite polynomials. Harmonic-oscillator wave functions. Correspondence with classical theory. Oscillating wave packet.

14 Spherically Symmetric Potentials in Three Dimensions 76
Separation of the wave equation. Legendre polynomials. Spherical harmonics. Parity. Angular momentum.

15 Three-dimensional Square Well Potential 83
Zero angular momentum. Interior solutions for arbitrary l. Exterior solutions for arbitrary l. Energy levels.

16 The Hydrogen Atom 88
Reduced mass. Asymptotic behavior. Energy levels. Laguerre polynomials. Hydrogen-atom wave functions. Degeneracy. Separation in parabolic coordinates. Energy levels. Wave functions.

Chapter 5 Continuous Eigenvalues: Collision Theory 100

17 One-dimensional Square Potential Barrier 101
Asymptotic behavior. Normalization. Scattering coefficients. Scattering of a wave packet.

18 Collisions in Three Dimensions 105
Scattering cross section. Relations between angles in the laboratory and center-of-mass systems. Relation between cross sections. Dependence on γ. Asymptotic behavior. Normalization.

19 Scattering by Spherically Symmetric Potentials 116
Asymptotic behavior. Differential cross section. Total elastic cross section. Phase shifts. Calculation of δ_l. Relation between signs of δ_l and $V(r)$. Ramsauer-Townsend effect. Scattering by a perfectly rigid sphere. Scattering by a square well potential. Resonance scattering. Angular distribution at low energies.

20 Scattering by Complex Potentials 129
Conservation of probability. Complex phase shifts. Asymptotic relations. Reciprocity theorem. Generalized optical theorem. Optical theorem.

21 Scattering by a Coulomb Field 138
Parabolic coordinates. Confluent hypergeometric function. Scattering cross section and normalization. Solution in spherical coordinates. Modified coulomb field. Classical limit for a pure coulomb field.

Chapter 6 Matrix Formulation of Quantum Mechanics 148

22 Matrix Algebra 149
Matrix addition and multiplication. Null, unit, and constant matrices. Trace, determinant, and inverse of a matrix. Hermitian and unitary matrices. Transformation and diagonalization of matrices. Functions of matrices. Matrices of infinite rank.

23 Transformation Theory 155
Unitary matrix W. Transformation of the hamiltonian with W. Transformation of the hamiltonian with U. Transformation of the hamiltonian with V. Representations of operators. A useful identity. Row and column matrices. Hilbert space. Dirac's bra and ket notation. Projection operators. Physical meaning of matrix elements.

24 Equations of Motion 167
Schrödinger picture. Heisenberg picture. Interaction picture. Energy representation. Classical lagrangian and hamiltonian equations of motion. Poisson brackets and commutator brackets.

Quantization of a classical system. Motion of a particle in an electromagnetic field. Evaluation of commutator brackets Velocity and acceleration of a charged particle. The Lorentz force. Virial theorem.

25 Matrix Theory of the Harmonic Oscillator 180
Energy representation. Raising and lowering operators. Matrices for a, x, and p. Coordinate representation.

Chapter 7 Symmetry in Quantum Mechanics 187

26 Space and Time Displacements 188
Unitary displacement operator. Equation of motion. Symmetry and degeneracy. Matrix elements for displaced states. The group concept. Time displacement.

27 Rotation, Angular Momentum, and Unitary Groups 194
Proper rotation group. Geometrical isomorphism. Infinitesimal rotations. Spin of a vector particle. Commutation relations for the generators. Choice of a representation. Values of m, $f(j)$, and λ_m. Angular momentum matrices. Connection with spherical harmonics. Spin angular momentum. Covering group. Unitary and special unitary groups in two dimensions. The groups $U(n)$ and $SU(n)$. Generators of $U(n)$ and $SU(n)$. The $SU(3)$ group. Representation in terms of coordinates and momenta.

28 Combination of Angular Momentum States and Tensor Operators 212
Eigenvalues of the total angular momentum. Clebsch-Gordan coefficients. Recursion relations. Construction procedure. Some particular coefficients. Matrix elements for rotated states. Irreducible tensor operators. Product of tensor operators. Combination of operator and eigenstate. Wigner-Eckart theorem.

29 Space Inversion and Time Reversal 224
Space inversion. Unitary inversion operator. Intrinsic parity. Inverted states and operators. Time reversal. Antilinear operators. Antiunitary operators. T for a zero spin particle. T for a nonzero spin particle. Systems of several particles. Reality of eigenfunctions.

30 Dynamical Symmetry 234
Classical Kepler problem. Hydrogen atom. The $O(4)$ group. Energy levels of hydrogen. Classical isotropic oscillator. Quantum isotropic oscillator.

Chapter 8 Approximation Methods for Bound States 244

31 Stationary Perturbation Theory 245
Nondegenerate case. First-order perturbation. Second-order perturbation. Perturbation of an oscillator. Degenerate case. Removal of degeneracy in second order. Zeeman effect without electron spin. First-order Stark effect in hydrogen. Perturbed energy levels. Occurrence of permanent electric dipole moments.

32 The Variation Method 255
Expectation value of the energy. Application to excited states. Ground state of helium. Electron interaction energy. Variation of the parameter Z. van der Waals interaction. Perturbation calculation. Variation calculation.

33 Alternative Treatment of the Perturbation Series 263
Second-order Stark effect in hydrogen. Polarizability of hydrogen. Method of Dalgarno and Lewis. Third-order perturbed energy. Interaction of a hydrogen atom and a point charge.

34 The WKB Approximation 268
Classical limit. Approximate solutions. Asymptotic nature of the solutions. Solution near a turning point. Linear turning point. Connection at the turning point. Asymptotic connection formulas. Energy levels of a potential well. A quantization rule. Special boundary conditions. Tunneling through a barrier.

35 Methods for Time-dependent Problems 279
Time-dependent perturbation theory. Interaction picture. First-order perturbation. Harmonic perturbation. Transition probability. Ionization of a hydrogen atom. Density of final states. Ionization probability. Second-order perturbation. Adiabatic approximation. Choice of phases. Connection with perturbation theory. Discontinuous change in H. Sudden approximation. Disturbance of an oscillator.

Chapter 9 Approximation Methods in Collision Theory 298

36 The Scattering Matrix 298
Green's functions and propagator. Free-particle Green's functions. Integral equation for ψ. Integral equation for the propagator. Use of the advanced Green's function. Differential equation for the Green's functions. Symbolic relations. Application to scattering. Unitarity of the S matrix. Symmetry properties of the S matrix.

37 Stationary Collision Theory 312
Transition matrix. Transition probability. Scattering cross section. Green's functions for stationary case. Green's functions as inverse operators. Stationary propagator. Free-particle propagator. Scattering amplitude. Ingoing waves. S matrix for stationary case. Angular momentum representation.

38 Approximate Calculations 324
Born approximation. Validity of the Born approximation. Scattering from two potentials. Distorted wave Born approximation. Partial wave analysis of the DWBA. Approximate expression for the phase shifts. Scatterer with internal degrees of freedom. Elastic and inelastic cross sections. Electron scattering from hydrogen. Production of a cloud chamber track. Second-order perturbation theory. Evaluation of the second-order matrix

element. Discussion of the cross section. Eikonal approximation. Scattering amplitude and cross section. Perfect absorber.

39 Analytic Properties and Dispersion Relations 344

Radial solutions. Jost function. Enhancement factor. Jost function for large $|k|$. Bound states. Dispersion relations for the Jost function. Dispersion relation for $\ln f_l(k)$. Effect of bound states. Levinson's theorem. Effective range. Forward scattering amplitude. Dispersion relation for $T(E)$. Subtracted dispersion relation.

Chapter 10 Identical Particles and Spin 362

40 Identical Particles 363

Physical meaning of identity. Symmetric and antisymmetric wave functions. Construction from unsymmetrized functions. The symmetric group. Distinguishability of identical particles. The exclusion principle. Connection with statistical mechanics. Collisions of identical particles.

41 Spin Angular Momentum 371

Connection between spin and statistics. Spin matrices and eigenfunctions. Collisions of identical particles. Electron spin functions. The helium atom. Spin functions for three electrons.

42 Density Operator and Density Matrix 378

Expectation value and projection operator. Density operator. Equations of motion. Projection operator for a spin $\frac{1}{2}$ particle. Density matrix for a spin $\frac{1}{2}$ particle. Polarization vector for a spin s particle. Precession of the polarization vector.

43 Rearrangement Collisions 384

Notation for rearrangement collisions. Alternative expression for the T matrix element. T matrix element for rearrangements. Presence of a core interaction. Elimination of the core term. Exchange collisions of electrons with hydrogen. Relation between amplitude and matrix element. Effects of identity and spin. Exchange collisions with helium.

Chapter 11 Semiclassical Treatment of Radiation 397

44 Absorption and Induced Emission 398

Maxwell's equations. Plane electromagnetic waves. Use of perturbation theory. Transition probability. Interpretation in terms of absorption and emission. Electric dipole transitions. Forbidden transitions.

45 Spontaneous Emission 406

Classical radiation field. Asymptotic form. Radiated energy. Dipole radiation. Angular momentum. Dipole case. Conversion from classical to quantum theory. Planck distribution formula. Line breadth.

46 Some Applications of Radiation Theory 416
Selection rules for a single particle. Polarization of emitted radiation. Conservation of angular momentum. Selection rules for many-particle systems. Photoelectric effect. Angular distribution. Cross section for the atomic photoelectric effect. Improvement on the Born approximation.

Chapter 12 Atoms, Molecules, and Atomic Nuclei 424

47 Approximations in Atomic Structure 424
Central-field approximation. Periodic system of the elements. Thomas-Fermi statistical model. Evaluation of the potential. Hartree's self-consistent fields. Connection with the variation method. Corrections to the central-field approximation. *LS* coupling scheme. Selection rules. *jj* coupling scheme.

48 The Alkali Atoms 436
Doublet separation. Doublet intensity. Effect of a magnetic field. Weak-field case. Strong-field case. Quadratic Zeeman effect.

49 Molecules 445
Classification of energy levels. Wave equation. The hydrogen molecule. Potential-energy function. The Morse potential. Rotation and vibration of diatomic molecules. Energy levels. Effect of nuclear identity.

50 Atomic Nuclei 455
General properties of nuclei. Two-nucleon interaction. Neutron-proton system. Arbitrary shape of potential. Relations for the phase shift. Effective range. Exchange operators. Proton-proton scattering.

Chapter 13 Relativistic Wave Equations 466

51 Schrödinger's Relativistic Equation 467
Free particle. Electromagnetic potentials. Separation of the equation. Energy levels in a coulomb field.

52 Dirac's Relativistic Equation 472
Free-particle equation. Matrices for α and β. Free-particle solutions. Charge and current densities. Electromagnetic potentials.

53 Dirac's Equation for a Central Field 480
Spin angular momentum. Approximate reduction; spin-orbit energy. Separation of the equation. The hydrogen atom. Classification of energy levels. Negative energy states.

Chapter 14 The Quantization of Wave Fields 490

54 Classical and Quantum Field Equations 491
Coordinates of the field. Time derivatives. Classical lagrangian equation. Functional derivative. Classical hamiltonian equations.

Quantum equations for the field. Fields with more than one component. Complex field.

55 Quantization of the Nonrelativistic Schrödinger Equation 498
Classical lagrangian and hamiltonian equations. Quantum equations. The N representation. Creation, destruction, and number operators. Connection with the many-particle Schrödinger equation. Anticommutation relations. Equation of motion. Physical implications of anticommutation. Representation of the anticommuting a_k operators.

56 Electromagnetic Field in Vacuum 508
Lagrangian equations. Hamiltonian equations. Quantum equations. Commutation relations for $\mathbf{E}$ and $\mathbf{H}$. Plane wave representation. Quantized field energy. Quantized field momentum. $\mathbf{A}(\mathbf{r},t)$ in the plane wave representation. Commutation relations at different times.

57 Interaction Between Charged Particles and the Electromagnetic Field 521
Lagrangian and hamiltonian equations. Elimination of ϕ. Quantization of the fields. Inclusion of static fields. Perturbation theory of the interparticle interaction. Einstein-Bose case. Fermi-Dirac case. Radiation theory. Transition probability for absorption. Transition probability for emission.

Index 535