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TABLE OF CONTENTS, designed as a coherent abstract in an attempt to provide the prospective reader with the orientation in which he will have to traverse the subject matter. This is followed by a	vii
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solely on analysis of experimental situations realized by selective measurements, with compound spin orientation experiments serving as illustration. One is led in this context to consider in

- \*Section 6 The Representation of Nonselective Measurements, requiring introduction of the concept of randomly distributed phases of measurement symbols. Once again idealized spin orientation measurements are used for illustration.
- Section 7 The Fundamental Dynamical Postulate governing the evolution of an object in time is stated in terms of the temporal development of expectation values, and from this formulation are derived state picture (or Schroedinger picture), operator picture (or Heisenberg picture), and interaction picture, together with the unitary transformations that interconnect them. The resulting dynamical equations are stated in both state vector and density matrix language, and solved by various methods for simple examples, the spin magnetic resonance problem being treated in detail. To enlarge the applicability of the concepts developed thus far, in
- Section 8 The Representation of Observables with Nondenumerably and Denumerably Infinite Ranges of Possible Values is sketched and the concept of the  $\psi$ -function of a state introduced, with emphasis on its geometrical interpretation as transformation function in Hilbert space. With this preparation one can now grasp in
- Section 9 Displacements of the Observer, leading in particular to an explicit representation of the Galileo transformation. By exploiting the correspondence of the resulting transformation formulae with the classical formulae, the momentum of an object is recognized as generator of the displacement operation, enabling one to derive the equation governing the  $\psi$ -function of an object in coordinate representation. The commutation relations between coordinate and momentum operators are found and used to introduce in
- \*Section 10 Uncertainties and the Relations between Them. Heisenberg's relation is derived, the concept of optimum state explained, and expressions for such states are calculated. The results of Section 9 enable one further to indulge by making in

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- \*Section II A Digression on Superfluidity, in which the concept of quasi particles is introduced phenomenologically, with an eye on a more rigorous derivation from particle theory given later in Section 30. The main thread of argument is then picked up again by considering in
- Section 12 Rotations of the Observer, leading, in analogy to the displacement operation, to recognition of the component operators of angular momentum as generators of rotations around corresponding axes. The problem of spin magnetic resonance is taken up once more and solved again, this time by transforming the dynamical equation to and from suitably chosen rotating coordinate frames. The fundamental theorem governing eigenstates of angular momentum is stated and its proof relegated to Appendix 1. By exhibiting in
- Section 13 The Connection between Invariance Properties of the Hamiltonian and Conservation Laws the ground is prepared for explaining in
- Section 14 The Invariance under Inversion of Coordinates and the Law of Conservation of Parity. The expectation value of any observable odd under inversion is shown to vanish in states of definite parity, and some physical consequences of this theorem are examined. The evidence gained from experiments involving the  $\beta$ -decay of nuclei is discussed in terms of nonconservation of parity, and the necessity of considering "combined inversion" as a symmetry operation is pointed out. By contrast, as shown in
- Section 15 Invariance under Reversal of Motion does not lead to a conserved quantum number analogous to parity, because of the anti-unitary nature of this so-called time reversal transformation. States can, however, be labeled by a dichotomic quantum number according to their behavior under repeated time reversal. The twofold degeneracy of all states that change sign under repeated time reversal is established, and the concept of the superselection rule is introduced. Eigenstates of angular momentum are used as illustration for both Sections 14 and 15.
  - Section 16 The Particle Concept in Quantum Mechanics is explained 115 as an abstraction far removed from the naive particle concept of classical physics, enabling one to grasp by introduction of

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the quasi-particle concept properties of macroscopic objects with the same formalism originally designed for description of so-called elementary particles. The division into fermions and bosons is defined. The exclusion principle is stated and used to describe in

Section 17 Fermion States in terms of a dichotomic occupation 119 number of quantum states, leading naturally to introduction of creation and annihilation operators and their representation. Neutrino and antineutrino states are used as simplest possible examples, and the possibility of understanding conservation of lepton numbers as superselection rules is pointed out. By an analogous development in

- Section 18 Boson States are introduced and illustrated by a derivation of the equations governing transverse photon states. Particular attention is given to the description of photon polarization, and by a formal generalization one is led to consider scalar and longitudinal photons. With this preparation in
- Section 19 Electrons and Positrons are treated as simplest examples 151 of fermions which do not have firm parallel or antiparallel alignment of spin with momentum because of their non-vanishing mass. The charge conjugation symmetry of the theory is exploited to circumvent introduction of negative energy states, and after positron states have been recognized as spatially inverted electron states the conservation of lepton number is once again viewed as a possible superselection rule. A procedure reducing four-component  $\psi$ -functions to a two-component form is explained.
- Section 20 The Lack of Sufficient Reason for Actually Existing Interactions is exhibited by recording the ambiguities inherent in attempts to account for the peculiar form of the electromagnetic interaction by making the phase of  $\psi$ -functions nonintegrable. The requirement of invariance under phase transformations leads one to consider in
- \*Section 21 The Idea of the Compensating Field, whose adoption requires recourse to vector mesons as the only primary agents of interaction between fermions. Attempts at removing inconsistencies encountered with interactions of short range are mentioned. Because of its long range one is tempted to view, as is done in

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	analogy to the gauge theory of electromagnetism, provided one takes into account the one basic difference between gravitation and electromagnetism, namely that inertial frames in presence of gravitation are, in general, accelerated with respect to each other if they are some finite distance apart.
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217	*Section 25 The Hierarchy of Propagators in presence of interaction all but forces one to view the world subspecie aeternitatis as an infinite concatenation of propagators, whose analytic penetration presents formidable difficulties which have as yet not been overcome. In absence of any internally consistent dynamical theory of interactions, one can fall back as in
233	Section 26 On Selection Rules Due to Symmetry under Inversions and Rotations of Coordinates. The system of two photons is treated as an example of how to determine the intrinsic parity of objects, such as positronium, for which this concept can be defined without ambiguity. The two-photon state is also particularly suited to illustrate, as is done in
241	Section 27 Permutation Symmetry of Multiple Particle States. Part of this section is devoted to construction of explicit representations for the operator of transposition of particle labels in angular momentum representation and in particle occupation number representation.
251	Section 28 Some Consequences of Symmetry under Particle Con- jugation and Time Reversal are derived by exploiting con-

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quantum number. The principle of reciprocity is proved and various conditions are examined under which this leads to validity of the principle of detailed balance.

- Section 29 Attributes Characteristic of Objects Engaging in Strong Interactions, such as isospin, baryon number, strangeness or hypercharge, require re-examination of the symmetry operations introduced in Sections 26, 27, 28, and lead to selection rules different from the ones encountered in the preceding sections. The section closes with an examination of all possible attributes conserved through superselection rules generated by anti-unitary symmetry operations.
- \*Section 30 The Quasi Particle Concept emerges as a powerful tool when one wishes to grasp the intrusion of quantum mechanical principles into the domain of macroscopic phenomena, such as superfluidity and superconductivity, encountered in substances at low temperatures. Some aspects of the quantum mechanical N-body problem, using the nonperturbative methods of Bogoliubov, Cooper, and Beliaev, are reviewed with the aim to exhibit the crucial part played by the principle of superposition of probability amplitudes in the minimization of energy through formation of quasi particles in presence of interactions between ordinary particles.
- Appendix | The Eigenstates of Angular Momentum are needed so often in the course of this work that their representation in both angular momentum space and coordinate space have been reviewed and collected in this appendix for ready reference. A similar need is accommodated by summarizing in
- Appendix 2 The Addition of Two Angular Momenta, containing tables for the Clebsch-Gordan coefficients for addition of an angular momentum  $j_2 = 1/2$  and an angular momentum  $j_2 = 1$ , respectively, to an arbitrary angular momentum  $j_1$ .
- Appendix 3 Vector Spherical Harmonics are defined and used for 323 classification of photon states, giving rise to additional information on the subjects treated in Sections 18 and 27.
- 327 Appendix 4 The Invariance of Dirac's Equation under Lorentz Transformations is treated mainly for the purpose of deriving a wellknown formula, governing the structure constants of this transformation, which is needed in Section 22.

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