

Contents

1. Introduction	1
1.1 The Charged Vacuum	1
1.2 From Theory to Experimental Verification	7
1.2.1 Superheavy Quasimolecules	8
1.2.2 Nuclear Sticking	9
1.2.3 <i>K</i> -Shell Ionization	11
1.3 Theoretical Developments	13
1.4 Historical Annotations on the Vacuum	14
1.4.1 The Concept of Vacuum	14
1.4.2 The Vacuum in Strong Fields	16
1.5 The Vacuum in Modern Quantum Physics	19
1.5.1 Pion Condensation	20
1.5.2 Strong Gravitational Fields	22
1.5.3 Vacuum Structure of Strongly Interacting Fermions and Bosons	24
Bibliographical Notes	25
2. The Wave Equation for Spin-1/2 Particles	26
2.1 The Dirac Equation	26
2.2 The Free Dirac Particle	30
2.3 Single-Particle Interpretation of Plane (Free) Dirac Waves	35
2.4 The Dirac Particle Coupled to Electromagnetic Fields – Non-Relativistic Limits and Spin of the Dirac Equation	36
2.5 Lorentz Covariance of the Dirac Equation	38
2.5.1 Formulation of Covariance (Form Invariance)	41
2.5.2 Determining the $\hat{S}(\hat{a})$ Operator for Infinitesimal Lorentz Transformations	46
2.5.3 The $\hat{S}(\hat{a})$ Operator for Finite Lorentz Transformations	47
2.5.4 Finite, Proper Lorentz Transformations	47
2.5.5 The \hat{S} Operator for Finite Lorentz Transformations	49
2.5.6 The Four-Current Density	51
2.6 Spinor Under Space Inversion (Parity Transformation)	52
2.7 Bilinear Covariants of Dirac Spinors	54
2.8 Gauge Invariant Coupling of Electromagnetic and Spinor Field	56
Bibliographical Notes	58

3. Dirac Particles in External Potentials	59
3.1 A Dirac Particle in a One-Dimensional Square Well Potential	59
3.2 A Dirac Particle in a Scalar, One-Dimensional Square Well Potential	67
3.3 A Dirac Particle in a Spherical Well	71
3.4 Solutions of the Dirac Equation for a Coulomb and a Scalar $1/r$ Potential	79
3.4.1 Pure Scalar Potential	83
3.4.2 Pure Coulomb Potential	84
3.4.3 Coulomb and Scalar Potential of Equal Strength ($\alpha' = \alpha''$) ..	86
3.5 Stationary Continuum Waves for a Dirac Particle in a Coulomb Potential	86
Bibliographical Notes	91
4. The Hole Theory	92
4.1 The “Dirac Sea”	92
4.1.1 Historical Context	95
4.2 Charge Conjugation Symmetry	96
4.3 Charge Conjugation of States in External Potential	102
4.3.1 Historical Note	104
4.4 Parity and Time-Reversal Symmetry	104
4.4.1 Parity Invariance	105
4.4.2 Time-Reversal Symmetry	106
Bibliographical Notes	111
5. The Klein Paradox	112
5.1 The Klein Paradox in the Single-Particle Interpretation of the Dirac Equation	112
5.2 Klein’s Paradox and Hole Theory	117
Bibliographical Notes	121
6. Resonant States in Supercritical Fields	122
6.1 Resonances in the Negative Energy Continuum	122
6.2 One Bound State Diving into One Continuum	123
6.2.1 Filled K Shell	132
6.2.2 Empty K Shell	135
6.3 Two and More Bound States Imbedded in One Continuum	137
6.4 One Bound State Imbedded in Several Continua	145
6.5 Overcritical Continuum States	149
6.5.1 Continuum Solutions for Extended Nuclei	150
6.5.2 Comments on the Point Nucleus Problem for $Z\alpha > \kappa $	154
6.5.3 The Physical Phase Shifts	158
6.5.4 Resonances in the Lower Continuum for $Z > Z_{\text{cr}}$	159
6.5.5 The Vacuum Charge Distribution	162
6.6 Some Useful Mathematical Relations	166
6.6.1 A Different Choice of Phases	172
Bibliographical Notes	173

7. Quantum Electrodynamics of Weak Fields	174
7.1 The Non-Relativistic Propagator	174
7.2 The <i>S</i> Matrix	177
7.3 Propagator for Electrons and Positrons	180
7.4 Relativistic Scattering Theory	188
Bibliographical Notes	193
8. The Classical Dirac Field Interacting with a Classical Electromagnetic Field – Formal Properties	194
8.1 Field Equations in Hamiltonian Form	194
8.2 Conservation Laws	197
8.3 Representation by Energy Eigenmodes	201
8.3.1 Time-Independent Potentials	202
8.3.2 Explicitly Time-Dependent Potentials	202
8.4 The Elementary Field Functions	206
Bibliographical Notes	211
9. Second Quantization of the Dirac Field and Definition of the Vacuum	212
9.1 Canonical Quantization of the Dirac Field	213
9.2 Fock Space and the Vacuum State (I)	218
9.3 Poincaré Invariance of the Quantum Theory	221
9.4 Gauge Invariance and Discrete Symmetries	225
9.5 The Vacuum State (II)	230
9.6 The Feynman Propagator	235
9.7 Charge and Energy of the Vacuum (I)	236
9.8 Charge and Energy of the Vacuum (II)	246
9.9 Appendix: Feynman Propagator for Time-Dependent Fields ...	251
Bibliographical Notes	256
10. Evolution of the Vacuum State in Supercritical Potentials	257
10.1 The In/Out Formalism	257
10.2 Evolution of the Vacuum State	260
10.3 Decay of a Supercritical <i>K</i> Vacancy – Projection Formalism ...	264
10.4 Decay of the Neutral Vacuum – Schrödinger Picture	273
10.5 The Vacuum in a Constant Electromagnetic Field	278
10.6 Quantum Electrodynamics in Strong Macroscopic Fields	289
10.7 Klein’s Paradox Revisited	293
Bibliographical Notes	299
11. Superheavy Quasimolecules	300
11.1 Heavy-Ion Collisions: General Remarks	300
11.2 The Two-Centre Dirac Equation	303
11.3 The Critical Distance R_{cr}	310
Bibliographical Notes	312

12. The Dynamics of Heavy-Ion Collisions	313
12.1 Rutherford Scattering	313
12.2 Expansion in the Quasi-Molecular Basis	315
12.3 Heavy-Ion Collisions: A Quantal Description	321
12.4 The Semiclassical Approximation	327
12.5 Collisions with Nuclear Interaction	330
12.6 Status of Numerical Calculations	339
Bibliographical Notes	344
13. Experimental Test of Supercritical Fields in Heavy-Ion Collisions	345
13.1 Establishing Superheavy Quasimolecules	345
13.2 Positron Spectrometers	356
13.2.1 The “Orange”-Type β Spectrometer	357
13.2.2 Solenoidal Transport Systems	359
13.3 Background Effects Creating Positrons	361
13.4 Positron Experiments I: Gross Features	364
13.5 Positron Experiments II: Deep Inelastic Collisions	370
13.6 Positron Experiments III: Narrow Structures in the Positron Spectrum	371
13.7 Giant Nuclear Systems and Spontaneous Positron Emission	378
Bibliographical Notes	388
14. Vacuum Polarization	389
14.1 Vacuum-Current Density: Perturbative Expansion	389
14.2 Gauge Invariance and Vacuum Polarization	395
14.3 Charge Renormalization	399
14.4 Explicit Form of the Polarization Function	402
14.5 Vacuum Polarization Effects in Atoms	409
Bibliographical Notes	414
15. Vacuum Polarization: Arbitrarily Strong External Potentials	415
15.1 Green’s Function for Arbitrarily Strong External Potentials	415
15.2 Vacuum Polarization Charge Density	420
15.3 Vacuum Polarization in External Fields of Arbitrary Strengths ..	424
15.4 Real and Virtual Vacuum Polarization	428
Bibliographical Notes	430
16. Many-Body Effects in QED of Strong Fields	431
16.1 Self-Consistent Hartree-Fock Equations	431
16.2 Self-Energy Effects in Atoms	436
16.3 Self-Energy in Superheavy Atoms	440
16.4 Supercharged Vacuum	445
16.5 The Problem of a Supercritical Point Charge	448
16.5.1 Overcritical Single-Particle States	450
16.5.2 Screening Effects of the Vacuum Charge	456
16.5.3 Influence of Heavier Leptons	465

16.6 Klein's Paradox Revisited	467
Bibliographical Notes	469
17. Bosons Bound in Strong Potentials	470
17.1 The Klein-Gordon Field	470
17.2 Alternate Form of the Klein-Gordon Equation	477
Bibliographical Notes	483
18. Subcritical External Potentials	484
18.1 Quantization of the Klein-Gordon Field with External Fields ...	484
18.2 (Quasi) Particle Representation of the Operators	486
18.3 The Fock Space and Diagonalization of the Hamiltonian	490
18.4 The Coulomb Problem for Spin-0 Particles	495
Bibliographical Notes	498
19. Overcritical Potential for Bose Fields	499
19.1 The Critical Potentials	499
19.2 The True Ground State and Bose Condensation	507
19.3 Solutions of the Condensate Equations	515
Bibliographical Notes	519
20. Strong Yang-Mills Fields	520
20.1 Quantum Chromodynamics	520
20.2 Gluon Condensates in Strong Colour Fields	528
20.3 The "Magnetic" Vacuum of QCD	534
20.4 Spontaneous Quark Pair Production and Fission of Quark Bags	540
Bibliographical Notes	549
21. Strong Fields in General Relativity	550
21.1 Dirac Particles in a Gravitational Field	550
21.2 Limiting Charge of Black Holes	557
21.3 Uniform Acceleration and Rindler Space	563
21.4 Event Horizon and Thermal Particle Spectrum	567
Bibliographical Notes	571
References	573
Subject Index	587