

---

# Contents

<b>1. Symmetries in Quantum Mechanics</b> .....	1
1.1 Symmetries in Classical Physics .....	1
1.2 Spatial Translations in Quantum Mechanics .....	18
1.3 The Unitary Translation Operator .....	19
1.4 The Equation of Motion for States Shifted in Space .....	20
1.5 Symmetry and Degeneracy of States .....	22
1.6 Time Displacements in Quantum Mechanics .....	30
1.7 Mathematical Supplement: Definition of a Group .....	32
1.8 Mathematical Supplement: Rotations and their Group Theoretical Properties .....	35
1.9 An Isomorphism of the Rotation Group .....	37
1.9.1 Infinitesimal and Finite Rotations .....	39
1.9.2 Isotropy of Space .....	41
1.10 The Rotation Operator for Many-Particle States .....	50
1.11 Biographical Notes .....	51
<b>2. Angular Momentum Algebra Representation of Angular Momentum Operators — Generators of SO(3).</b> .....	53
2.1 Irreducible Representations of the Rotation Group .....	53
2.2 Matrix Representations of Angular Momentum Operators ..	57
2.3 Addition of Two Angular Momenta .....	66
2.4 Evaluation of Clebsch-Gordan Coefficients. ....	70
2.5 Recursion Relations for Clebsch-Gordan Coefficients .....	71
2.6 Explicit Calculation of Clebsch-Gordan Coefficients .....	72
2.7 Biographical Notes .....	79
<b>3. Mathematical Supplement: Fundamental Properties of Lie Groups</b> ..	81
3.1 General Structure of Lie Groups .....	81
3.2 Interpretation of Commutators as Generalized Vector Products, Lie's Theorem, Rank of Lie Group .....	91
3.3 Invariant Subgroups, Simple and Semisimple Lie Groups, Ideals .....	93
3.4 Compact Lie Groups and Lie Algebras .....	101
3.5 Invariant Operators (Casimir Operators) .....	101
3.6 Theorem of Racah .....	102
3.7 Comments on Multiplets .....	102
3.8 Invariance Under a Symmetry Group .....	104

3.9	Construction of the Invariant Operators	108
3.10	Remark on Casimir Operators of Abelian Lie Groups	110
3.11	Completeness Relation for Casimir Operators	110
3.12	Review of Some Groups and Their Properties	112
3.13	The Connection Between Coordinate Transformations and Transformations of Functions	113
3.14	Biographical Notes	126
<b>4.</b>	<b>Symmetry Groups and Their Physical Meaning</b>	
	–General Considerations	127
4.1	Biographical Notes	132
<b>5.</b>	<b>The Isospin Group (Isobaric Spin)</b>	133
5.1	Isospin Operators for a Multi-Nucleon System	139
5.2	General Properties of Representations of a Lie Algebra	146
5.3	Regular (or Adjoint) Representation of a Lie Algebra	148
5.4	Transformation Law for Isospin Vectors	152
5.5	Experimental Test of Isospin Invariance	159
5.6	Biographical Notes	174
<b>6.</b>	<b>The Hypercharge</b>	175
6.1	Biographical Notes	181
<b>7.</b>	<b>The SU(3) Symmetry</b>	183
7.1	The Groups $U(n)$ and $SU(n)$	183
	7.1.1. The Generators of $U(n)$ and $SU(n)$	185
7.2	The Generators of $SU(3)$	187
7.3	The Lie Algebra of $SU(3)$	190
7.4	The Subalgebras of the $SU(3)$ -Lie Algebra and the Shift Operators	198
7.5	Coupling of $T$ -, $U$ - and $V$ -Multiplets	201
7.6	Quantitative Analysis of Our Reasoning	202
7.7	Further Remarks About the Geometric Form of an $SU(3)$ Multiplet	204
7.8	The Number of States on Mesh Points on Inner Shells	205
<b>8.</b>	<b>Quarks and SU(3)</b>	217
8.1	Searching for Quarks	219
8.2	The Transformation Properties of Quark States	220
8.3	Construction of all $SU(3)$ Multiplets from the Elementary Representations $[3]$ and $[\bar{3}]$	226
8.4	Construction of the Representation $D(p, q)$ from Quarks and Antiquarks	228
	8.4.1. The Smallest $SU(3)$ Representations	231
8.5	Meson Multiplets	240
8.6	Rules for the Reduction of Direct Products of $SU(3)$ Multiplets	244

8.7	<i>U</i> -spin Invariance .....	248
8.8	Test of <i>U</i> -spin Invariance .....	250
8.9	The Gell-Mann-Okubo Mass Formula .....	252
8.10	The Clebsch-Gordan Coefficients of the SU(3) .....	254
8.11	Quark Models with Inner Degrees of Freedom .....	257
8.12	The Mass Formula in SU(6) .....	283
8.13	Magnetic Moments in the Quark Model .....	284
8.14	Excited Meson and Baryon States .....	286
	8.14.1 Combinations of More Than Three Quarks .....	286
8.15	Excited States with Orbital Angular Momentum .....	288
<b>9.</b>	<b>Representations of the Permutation Group and Young Tableaux ..</b>	<b>291</b>
9.1	The Permutation Group and Identical Particles .....	291
9.2	The Standard Form of Young Diagrams .....	295
9.3	Standard Form and Dimension of Irreducible Representations of the Permutation Group $S_N$ .....	297
9.4	The Connection Between SU(2) and $S_2$ .....	307
9.5	The Irreducible Representations of SU( $n$ ) .....	310
9.6	Determination of the Dimension .....	316
9.7	The SU( $n - 1$ ) Subgroups of SU( $n$ ) .....	320
9.8	Decomposition of the Tensor Product of Two Multiplets ..	322
<b>10.</b>	<b>Mathematical Excursion. Group Characters .....</b>	<b>327</b>
10.1	Definition of Group Characters .....	327
10.2	Schur's Lemmas .....	328
	10.2.1 Schur's First Lemma .....	328
	10.2.2 Schur's Second Lemma .....	328
10.3	Orthogonality Relations of Representations and Discrete Groups .....	329
10.4	Equivalence Classes .....	331
10.5	Orthogonality Relations of the Group Characters for Discrete Groups and Other Relations .....	334
10.6	Orthogonality Relations of the Group Characters for the Example of the Group $D_3$ .....	334
10.7	Reduction of a Representation .....	336
10.8	Criterion for Irreducibility .....	337
10.9	Direct Product of Representations .....	337
10.10	Extension to Continuous, Compact Groups .....	338
10.11	Mathematical Excursion: Group Integration .....	339
10.12	Unitary Groups .....	340
10.13	The Transition from U( $N$ ) to SU( $N$ ) for the Example SU(3) .....	342
10.14	Integration over Unitary Groups .....	344
10.15	Group Characters of Unitary Groups .....	347
<b>11.</b>	<b>Charm and SU(4) .....</b>	<b>365</b>
11.1	Particles with Charm and the SU(4) .....	367

11.2	The Group Properties of SU(4) .....	367
11.3	Tables of the Structure Constants $f_{ijk}$ and the Coefficients $d_{ijk}$ for SU(4) .....	376
11.4	Multiplet Structure of SU(4) .....	378
11.5	Advanced Considerations .....	385
11.5.1	Decay of Mesons with Hidden Charm .....	385
11.5.2	Decay of Mesons with Open Charm .....	386
11.5.3	Baryon Multiplets .....	387
11.6	The Potential Model of Charmonium .....	398
11.7	The SU(4) [SU(8)] Mass Formula .....	406
11.8	The $\Upsilon$ Resonances .....	409
<b>12.</b>	<b>Mathematical Supplement</b> .....	<b>413</b>
12.1	Introduction .....	413
12.2	Root Vectors and Classical Lie Algebras .....	417
12.3	Scalar Products of Eigenvalues .....	421
12.4	Cartan-Weyl Normalization .....	424
12.5	Graphic Representation of the Root Vectors .....	424
12.6	Lie Algebra of Rank 1 .....	425
12.7	Lie Algebras of Rank 2 .....	426
12.8	Lie Algebras of Rank $l > 2$ .....	426
12.9	The Exceptional Lie Algebras .....	427
12.10	Simple Roots and Dynkin Diagrams .....	428
12.11	Dynkin's Prescription .....	430
12.12	The Cartan Matrix .....	432
12.13	Determination of all Roots from the Simple Roots .....	433
12.14	Two Simple Lie Algebras .....	435
12.15	Representations of the Classical Lie Algebras .....	436
<b>13.</b>	<b>Special Discrete Symmetries</b> .....	<b>441</b>
13.1	Space Reflection (Parity Transformation) .....	441
13.2	Reflected States and Operators .....	443
13.3	Time Reversal .....	444
13.4	Antiunitary Operators .....	445
13.5	Many-Particle Systems .....	450
13.6	Real Eigenfunctions .....	451
<b>14.</b>	<b>Dynamical Symmetries</b> .....	<b>453</b>
14.1	The Hydrogen Atom .....	453
14.2	The Group SO(4) .....	455
14.3	The Energy Levels of the Hydrogen Atom .....	456
14.4	The Classical Isotropic Oscillator .....	458
14.4.1	The Quantum Mechanical Isotropic Oscillator .....	458

---

<b>15. Mathematical Excursion: Non-compact Lie Groups</b> .....	473
15.1 Definition and Examples of Non-compact Lie Groups .....	473
15.2 The Lie Group $SO(2,1)$ .....	480
15.3 Application to Scattering Problems .....	484
<b>Subject Index</b> .....	489